education
Department of Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

GRADE 12

MOCK EXAM

PAPER 2

18 AUGUST 2023

MARKS: 150

DURATION: 3 HOURS

This question paper consists of 15 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless otherwise stated.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise

QUESTION 1

The speeds, in kilometres per hour, of cyclists that passed a point on the route of the Ironman Race were recorded and summarised in the table below:

Speed (km/h)	Frequency (f)	Cumulative Frequency
$0 < x \leq 10$	10	10
$10 < x \leq 20$		30
$20 < x \leq 30$	45	
$30 < x \leq 40$	72	
$40 < x \leq 50$		170

1.1 Complete the above table in the ANSWER BOOK provided. (2)

1.2 Make use of the axes provided in the ANSWER BOOK to draw a cumulative frequency curve for the above data. (3)

1.3 Indicate clearly on your graph where the estimates of the lower quartile (Q1) and median (M) speeds can be read off. Write down these estimates. (2)

1.4 Draw a box and whisker diagram for the data. Use the number line in the ANSWER BOOK. (2)

1.5 Use your graph to estimate the number of cyclists that passed the point with speeds greater than 35 km/h. (1)

[10]

QUESTION 2

During the month of June patients visited a number of medical facilities for treatment.

The table shows the number of patients treated on certain dates during the month of June

Dates in the month of June	3	5	8	12	15	19	22	26
Number of patients treated	270	275	376	420	602	684	800	820

2.1 On **DIAGRAM SHEET 2**, draw a scatter plot of the given data. (3)

2.2 Determine the equation of the least squares regression line of patients treated (y) against date (x). (3)

2.3 Estimate how many patients have been treated on the 24th of June. (2)

2.4 Draw the least squares regression line on the grid on DIAGRAM SHEET 2. (3)

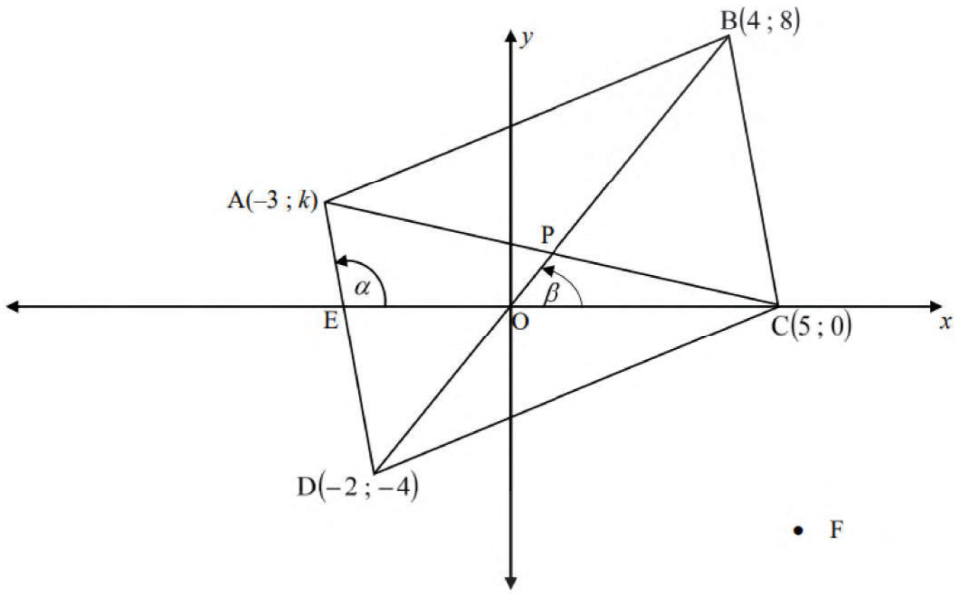
2.5 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (3)

2.6 Given that the mean for patients treated on certain dates is 528,63 calculate how many patients were within one deviation of the mean (3)

[17]

QUESTION 3

In the diagram below, A (–3 ; k), B(4 ; 8), C(5 ; 0) and D (–2 ; –4) are vertices of the parallelogram ABCD. Diagonals AC and BD bisect each other at P. The angles of inclination of AD and BD are α and β respectively. AD cuts the x-axis at E. F is a point in the fourth quadrant.



3.1 Determine the gradient of BC. (2)

3.2 If the distance between points A(–3 ; k) and B(4 ; 8) is 65 , calculate the value of k. (4)

3.3 Prove, using analytical geometry methods, that BP ⊥ AC. (3)

3.4 Calculate the coordinates of F if it is given that ACFD is a parallelogram. (2)

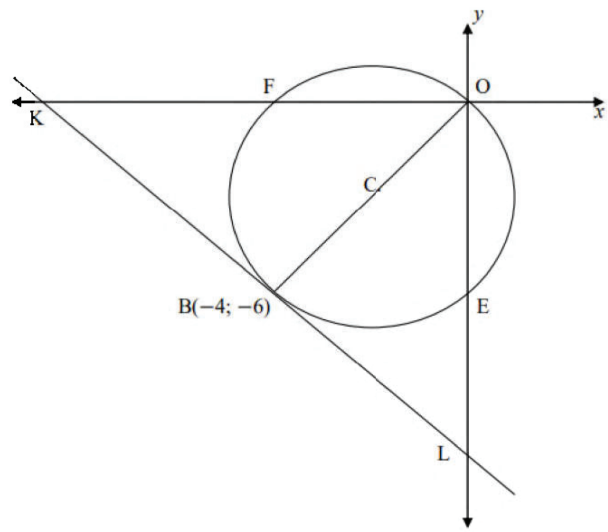
3.5 Calculate the size of $E\hat{D}O$ (correct to ONE decimal place). (6)

3.6 Calculate the area of ΔADC. (4)

[21]

QUESTION 4

4. A circle with centre at C passes through the origin, O, and also intersects the x-axis at F and the y-axis at E. The tangent to the circle at B (4 ; 6) intersects the x-axis at K and the y-axis at L.



4.1	Calculate the length of the radius of the circle.	(3)
4.2	Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$.	(4)
4.3	What type of a triangle is ΔOBL ? Give reason for your answer.	(2)
4.4	Determine the equation of the tangent KL.	(4)
4.5	Determine the co-ordinates of E.	(2)
4.2.6	Determine whether EF is a diameter of the circle. Show all working.	(5)

[20]

QUESTION 5

5.1 If $\tan 58° = m$, determine the following in terms of m without using a calculator.

5.1.1	$\sin 58°$	(2)
5.1.2	$\sin 296°$	(3)
5.1.3	$\cos 2°$	(3)

5.2 If $5\tan\theta + 2\sqrt{6} = 0$ and $0° < \theta < 270°$, determine with the aid of a sketch and without using the calculator, the value of :

5.2.1 $\sin\theta$ (2)

5.2.2 $\cos\theta$ (1)

5.1.3 $\dfrac{14\cos\theta + 7\sqrt{6}\sin\theta}{\cos(-240°).\tan 225°}$ (4)

[15]

QUESTION 6

5.1 Determine the value of $\dfrac{\cos(180° + x).\tan(360° - x).\sin^2(90° - x)}{\sin(180° - x)} + \sin^2 x$ (6)

5.2.1 Prove the identity: $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$ (3)

5.2.2 Hence calculate, without using a calculator, the value of

$\cos 15° - \cos 75°$ (4)

5.3 Find the value of $\tan\theta$, if the distance between $A(\cos\theta;\ \sin\theta)$ and $B(6;\ 7)$ is $\sqrt{86}$. (4)

[17]

QUESTION 7

Consider: $f(x) = \cos(x - 45°)$ and $f(x) = \tan\frac{1}{2}x$ for $x \in [-180°\ ;\ 180°]$

6.1 Use the grid provided to draw sketch graphs of f and g on the same set of axes for $x \in [-180°\ ;\ 180°]$. Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)

6.2 Use your graphs to answer the following questions for $x \in [-180°\ ;\ 180°]$

6.2.1 Write the solutions of $\cos(x - 45°) = 0$ (2)

6.2.2 Write down the equations of asymptote(s) of g. (2)

6.2.3 Write down the range of f. (1)

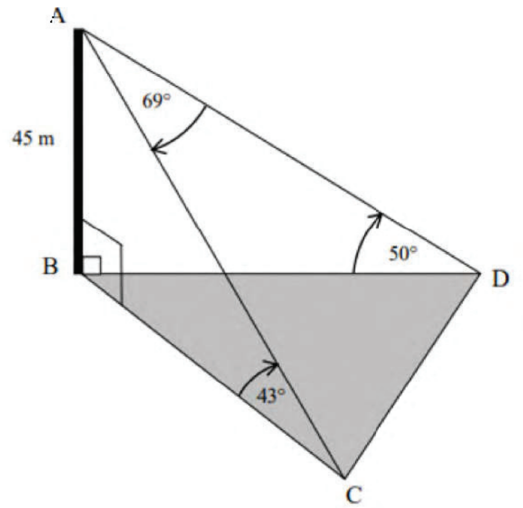
6.2.4 How many solutions exist for the equation $\cos(x - 45°) = \tan\frac{1}{2}x$? (1)

6.2.5 For what value(s) of x is $f(x).g(x) > 0$? (3)

[15]

QUESTION 7

In the figure below Thabo is standing at a point A on top of building AB that is 45 m high. He observes two cars at C and D respectively. The cars at C and D are in the same horizontal plane as B. The angle of elevation from C to A is 43° and the angle of elevation from D to A is 50° and $C\hat{A}D = 69°$

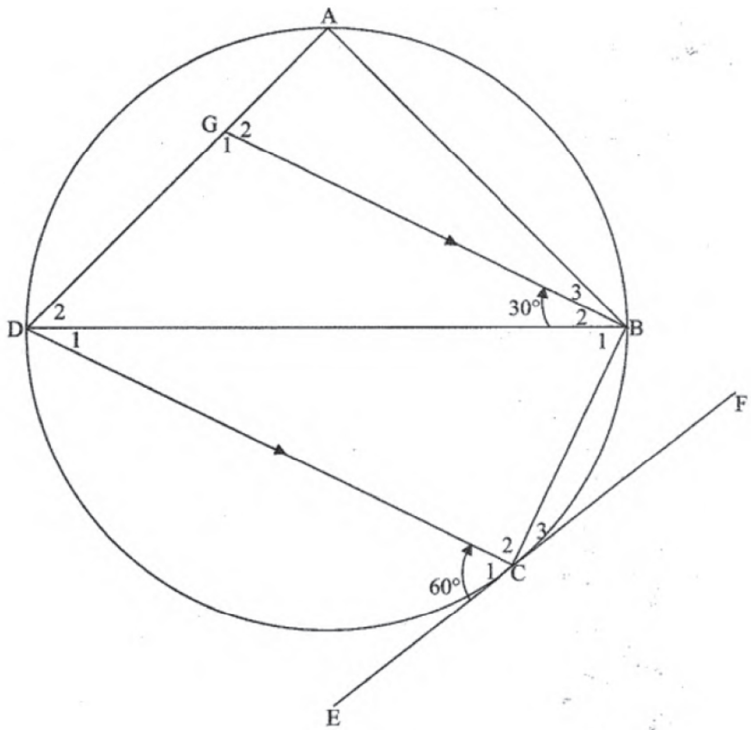


7.1	Calculate the lengths of AC and AD, correct to 2 decimal places.	(4)
7.2	Calculate the distance between the two cars, the length of CD.	(3)
		[7]

QUESTION 8

PROVIDE REASONS FOR ALL YOUR STATEMENTS AND CALCULATIONS IN QUESTION 8, 9 AND 10

In the diagram, ABCD is a cyclic quadrilateral. G is a point on AD such that BG || CD. ECF is a tangent to the circle at C. BD is a chord of the circle. $G\hat{B}D = 30°$ and $D\hat{C}E = 60°$



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{D}_1$ (1)

8.1.2 $\hat{B}_1$ (2)

8.1.3 $\hat{C}_2$ (1)

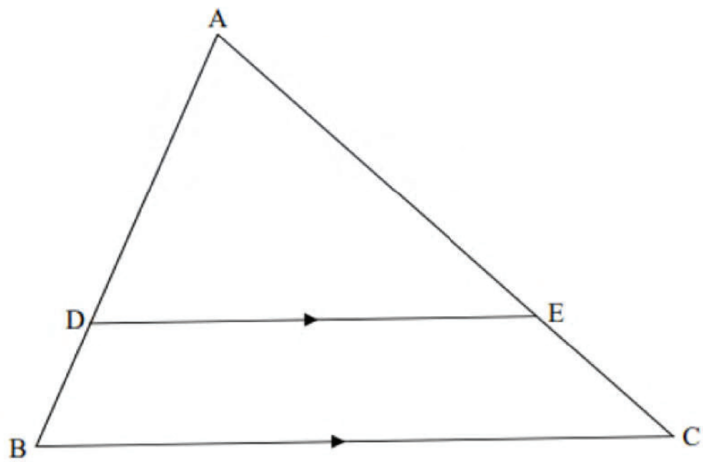
8.1.4 $D\hat{A}B$ (2)

8.2 Is BD a diameter of the circle? Motivate your answer. (2)

[8]

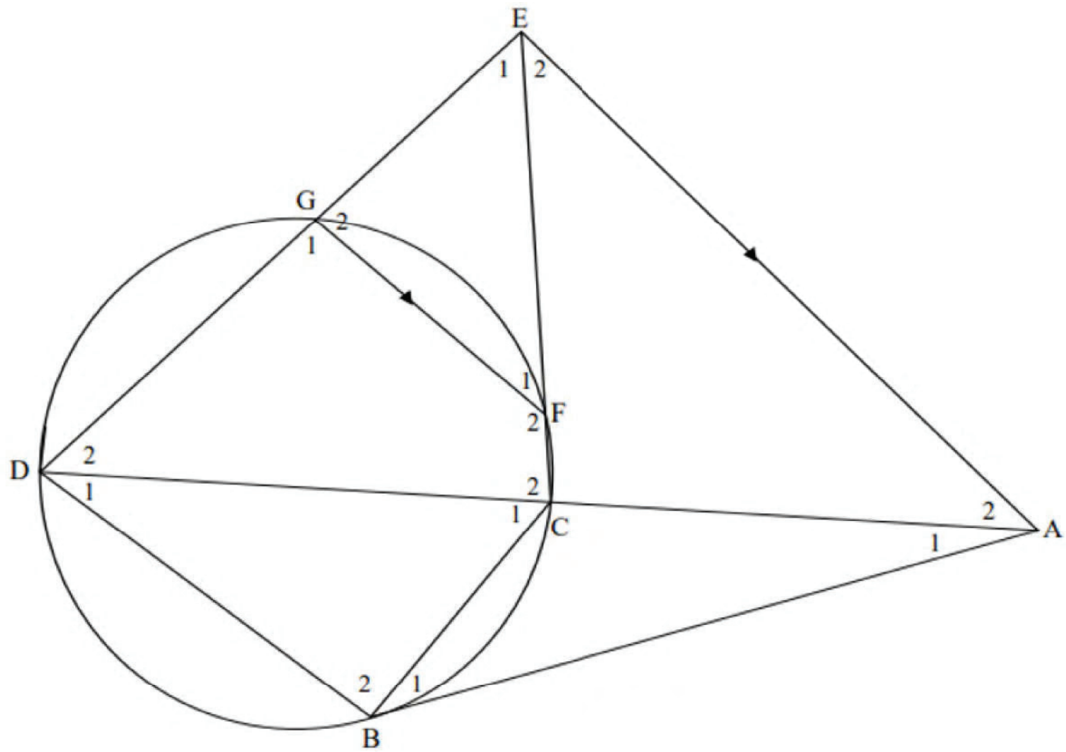
QUESTION 9

9.1 In ΔABC below, D and E are points on AB and AC respectively such that DE || BC. Prove the theorem that states that $\frac{AD}{DB}=\frac{AE}{EC}$ (6)



9.2 In the diagram below, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. 2 Chords BD and BC are drawn. DG and CF produced meet at E and DC is produced to A.

EA | |GF.



9.2.1 Give a reason why $\hat{B}_1 = \hat{D}_1$ (1)

9.2.2 Prove that $\Delta ABC ||| \Delta ADB$. (3)

9.2.3 Prove $\hat{E}_2 = \hat{D}_2$ (4)

9.2.4 Prove $AE^2 = AD \times AC$ (4)

9.2.5 Hence, deduce that AE=AB (3)

[21]

SCHOOL NAME: ______________________________

Name :_______________________________________ **Grade 12**___________

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QUESTION 1.2

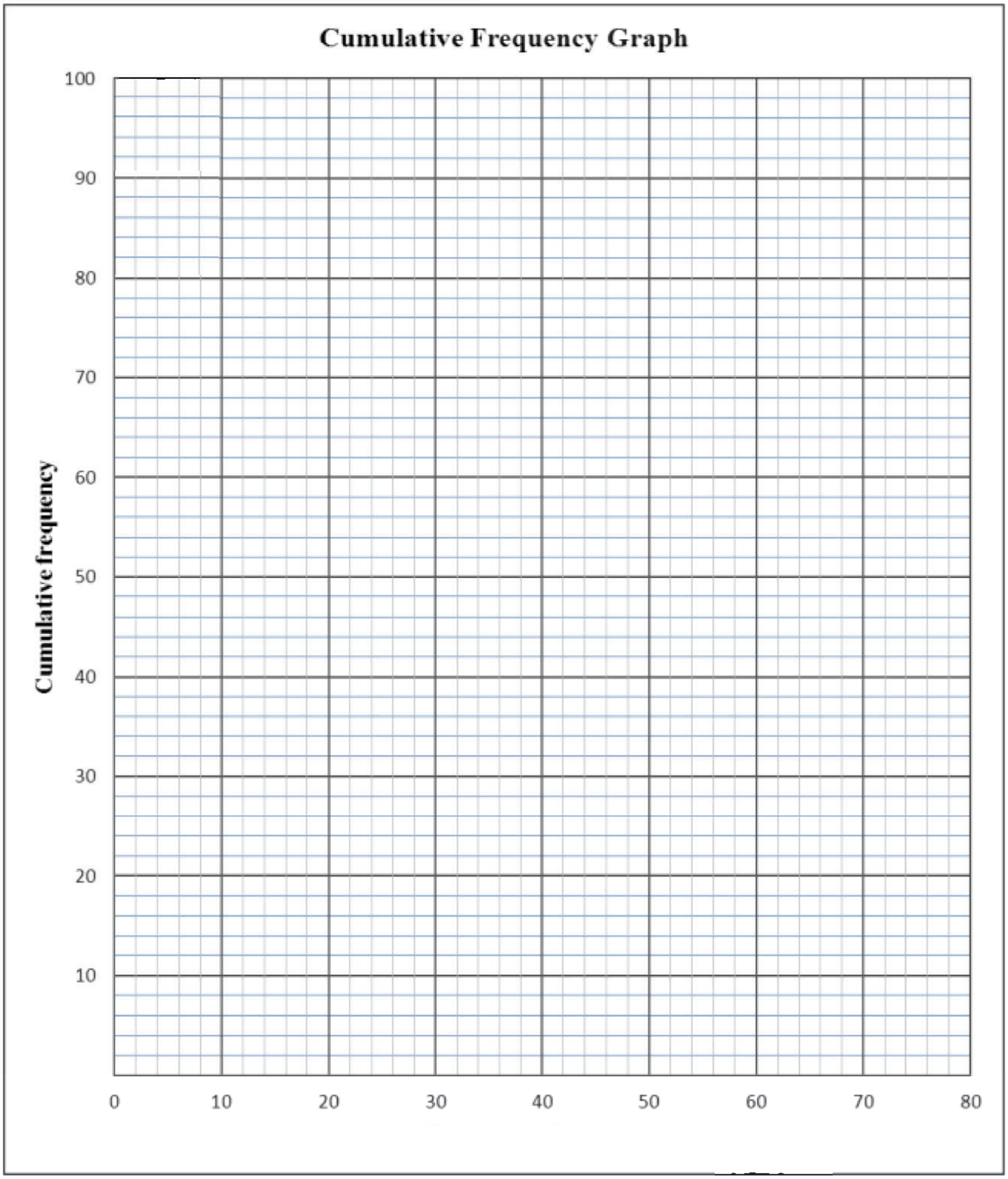
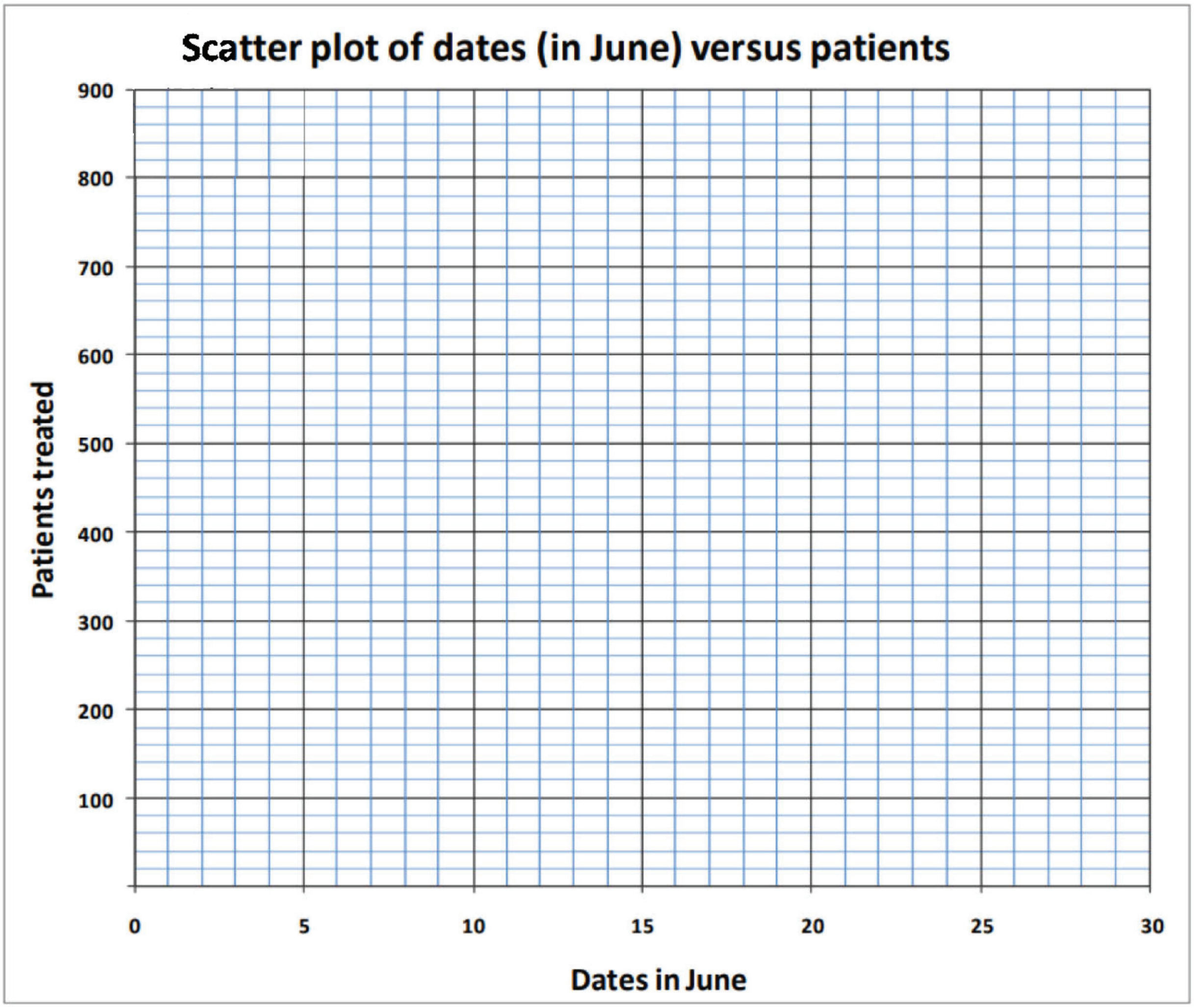
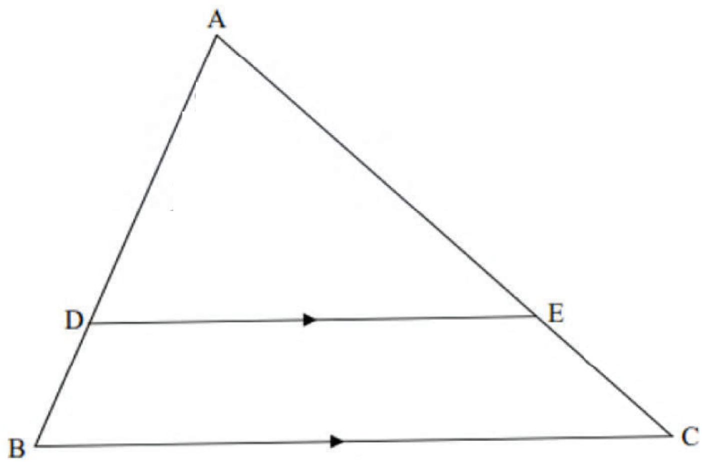

Cumulative Frequency Graph
Cumulative frequency
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80

DIAGRAM SHEET 2

QUESTIONS 2.1 AND 2.4



QUESTION 9.1



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$