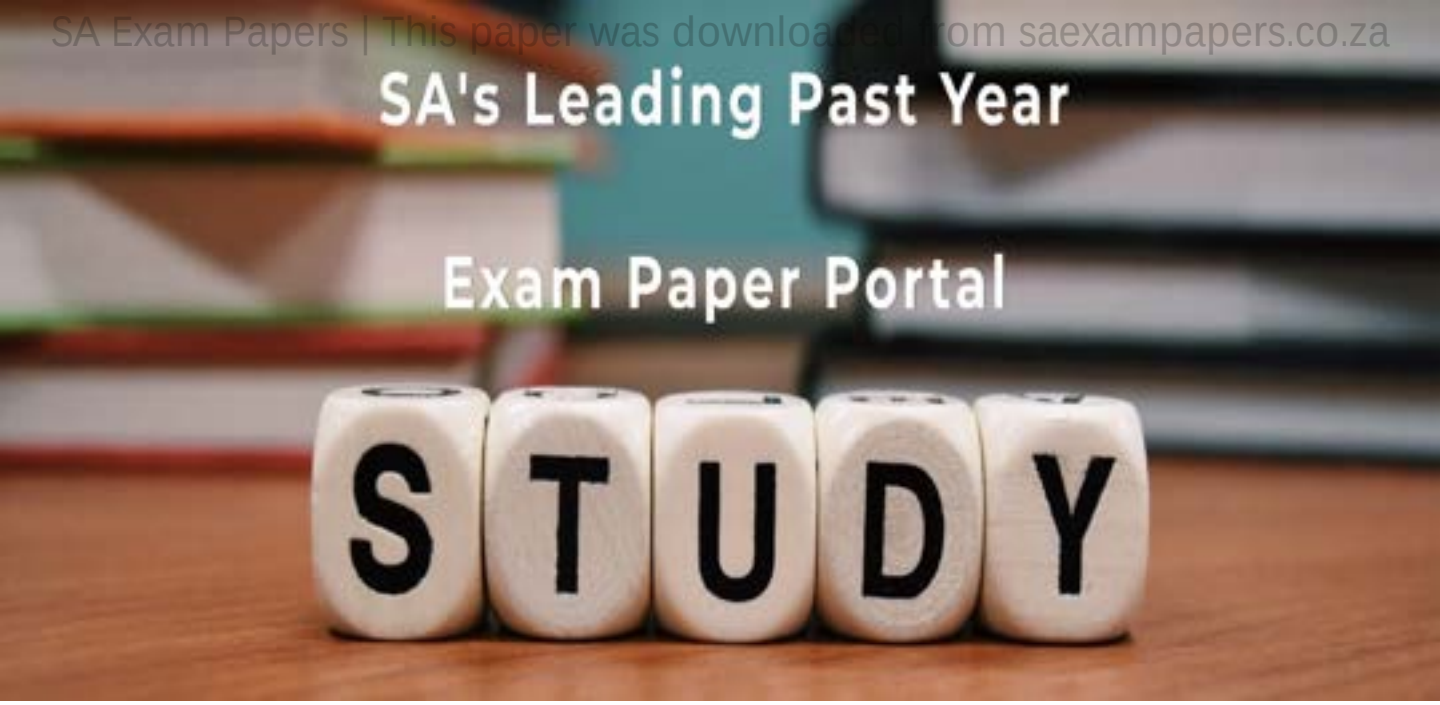


basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P1

2023

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 4.3.3 and QUESTION 7.5 on the ANSWER SHEETS provided. Write your centre number and examination number in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $\frac{1}{2}x(2x-1)=0$ (2)

1.1.2 $-x(6-x)=4$ (round off to TWO decimal places) (4)

1.1.3 $(2-x)(x+5)>0$ (2)

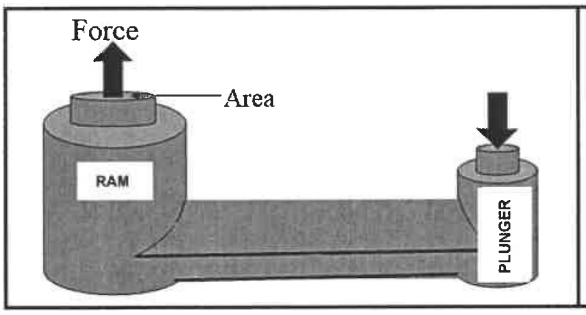
1.2 Given: $y+x-10=0$ and $x^2-xy+y^2=28$

1.2.1 Express $y+x-10=0$ in the form $y=mx+c$ (1)

1.2.2 Hence, or otherwise, solve for x and y. (6)

1.3 The formula used to determine the pressure of the fluid in a hydraulic system, as shown in the diagram below, is given by:

$$P=\frac{F}{A}$$



P = Pressure of the fluid in Pascal (Pa)

F = Force in Newton (N)

A = Area in square metres (m^2)

1.3.1 Make A the subject of the formula. (1)

1.3.2 Hence, or otherwise, calculate the value of A if P = 25 984 480,5 Pa and $F=25\times10^3$ N. Express the value in scientific notation. (3)

1.4 Given: $A=1000111_2$ and $B=10011_2$

1.4.1 Determine the value of A – B (in binary form). (2)

1.4.2 Hence, convert the answer to QUESTION 1.4.1 to decimal form. (1)
[22]

QUESTION 2

2.1 Given: $q = \dfrac{3 \pm \sqrt{1-3k}}{k-4}$

Determine for which value(s) of k will q:

2.1.1 Have equal roots (1)

2.1.2 Be undefined (1)

2.2 Given the equation: $4x^2 + 3x + p = 0$

2.2.1 Complete the following statement:

If the roots are non-real, then the value of the discriminant is … (1)

2.2.2 Determine the value of p, for which the roots of the equation will be non-real. (4)
[7]

QUESTION 3

3.1 Simplify the following, **without using a calculator**:

3.1.1 $7(3x)^0$ (1)

3.1.2 $\sqrt{8}\left(\sqrt{242} - \sqrt{72}\right)$ (3)

3.1.3 $\dfrac{9^{n-1} \times 27^{3-2n}}{81^{2-n}}$ (4)

3.2 Solve for x: $\log(x+2) - \log x = 1$ (4)

3.3 Given the complex number: $z = 5 - 5i$

3.3.1 Write down the quadrant, in the complex plane, in which z lies. (1)

3.3.2 Express the complex number z in polar form. (4)

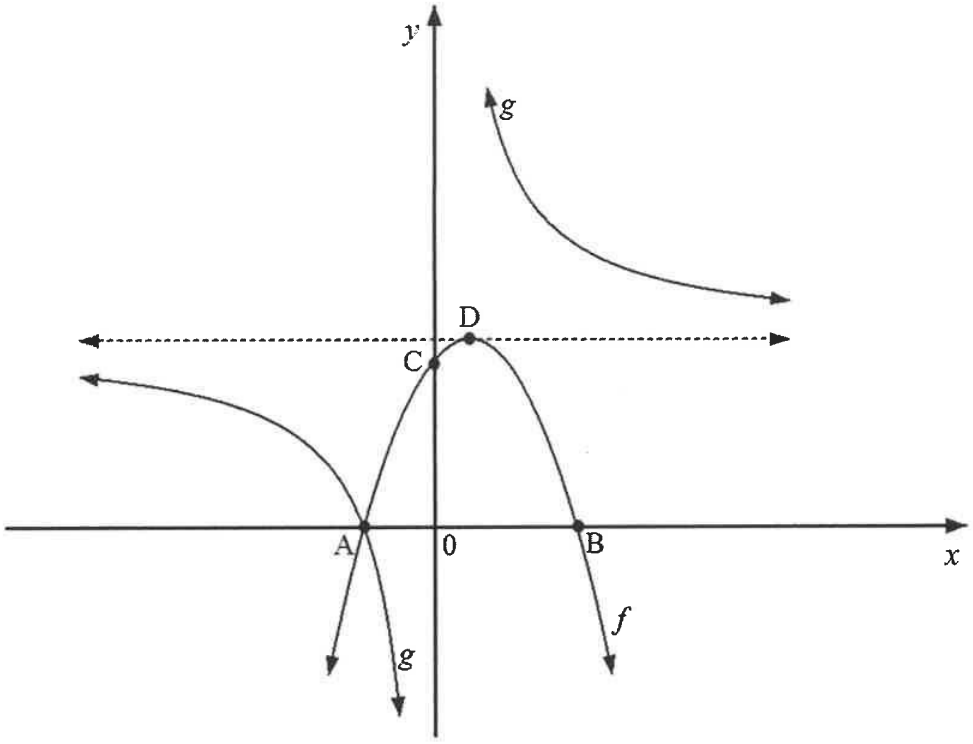
3.4 Solve for m and n if $m = 3i(2i-5) + 7 - ni$ (4)
[21]

QUESTION 4

4.1 Sketched below are the graphs of the functions defined by:

$f(x) = -(x-4)(x+2)$ and $g(x) = \frac{k}{x} + q$

- A, B and C are the intercepts of f.
- A is also the x-intercept of g.
- D is the turning point of f and lies on the asymptote of g.



4.1.1 Write down the x-coordinates of A and B. (2)

4.1.2 Determine the coordinates of D. (3)

4.1.3 Write down:

(a) The range of f (1)

(b) The equations of the asymptotes of g (2)

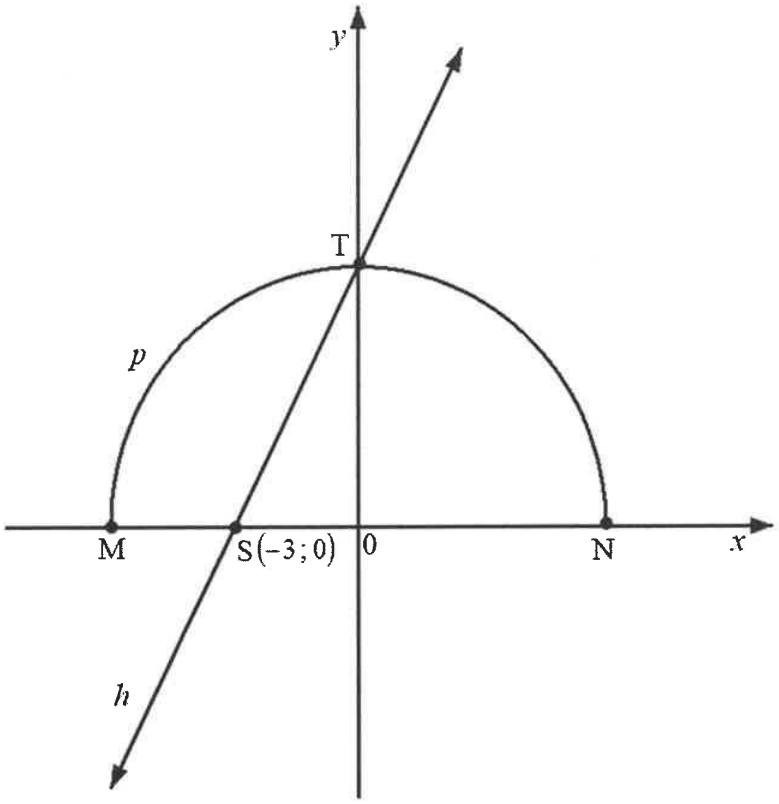
4.1.4 Determine the numerical value of k. (3)

4.1.5 Determine the values of x for which $g(x) \leq 0$ (2)

4.2 Sketched below are the graphs of functions defined by:

$h(x) = 2x + c$ and $p(x) = \sqrt{r^2 - x^2}$

- M, N and T are the intercepts of p.
- T and S $(-3\ ;\ 0)$ are the intercepts of h.



4.2.1 Determine the numerical value of c. (2)

4.2.2 Write down:

(a) The coordinates of M (1)

(b) The equation of p (1)

(c) The domain of p (2)

4.3 Given function k defined by $k(x) = \left(\frac{1}{3}\right)^x - 9$

4.3.1 Write down the equation of the asymptote of k. (1)

4.3.2 Determine the x-intercept and the y-intercept of k. (3)

4.3.3 Sketch the graph of k on the ANSWER SHEET provided. Clearly show the intercepts with the axes and the asymptote. (3)

[26]

QUESTION 5

5.1 A car's engine overheated at an increasing compound rate of 26% per hour due to a leak in one of the water pipes. The initial temperature before the engine overheated was 75 °C.

Determine the temperature of the engine at the end of 3 hours if it continued to overheat at the same rate. (3)

5.2 An engineering company purchased a printer for R81 000. The value of the printer depreciated at a rate of 20% per annum using the reducing-balance method.

5.2.1 Write down the value of the printer if it depreciated to $\frac{1}{3}$ of its original value. (1)

5.2.2 Determine how long it will take for the printer to depreciate to $\frac{1}{3}$ of its original value. (4)

5.3 A municipality plans to upgrade a clinic in 5 years' time, starting on 1 February 2028. The upgrade cost will be R95 000.

The municipality plans to invest in a savings account as follows:

- R30 000 is to be invested on 1 February 2023.
- A further R20 000 is to be invested on 1 February 2025.
- Another amount of R10 000 is to be invested on 1 February 2026.
- The interest rate for the first two years will be 10% per annum, compounded monthly.
- Thereafter, the interest rate will change to 12% per annum, compounded semi-annually.

Determine whether the municipality would be able to save enough money for the clinic upgrade at the end of the 5-year investment period. (6)
[14]

QUESTION 6

6.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x)=\frac{7}{2}x+5$ (5)

6.2 Determine:

6.2.1 $f'(x)$ if $f(x)=-8\pi$ (1)

6.2.2 $\frac{dy}{dx}$ if $y=\frac{x^4+9x}{x^2}$ (3)

6.2.3 $D_x\left[\left(\sqrt{x}+8x\right)^2\right]$ (5)

6.3 Given: $g(x)=3x^2+9x$

6.3.1 Determine $g'(x)$ (1)

6.3.2 Determine the gradient of the tangent to the curve of g at the point where $x=-3$ (2)

6.3.3 Hence, determine the equation of the tangent to the curve of g where $x=-3$ (3)

[20]

QUESTION 7

Given a function defined by $f(x)=-x^3+6x^2-3x-10=-(x-2)(x^2-4x-5)$

7.1 Write down the coordinates of the y-intercept of f. (2)

7.2 Show that $(x+1)$ is a factor of f. (2)

7.3 Hence, determine the x-intercepts of f. (2)

7.4 Determine the coordinates of the turning points of f. (5)

7.5 Sketch the graph of f on the ANSWER SHEET provided. (4)

7.6 Use your graph to write down the values of x if $x\cdot f'(x)>0$ and $x>0$ (2)

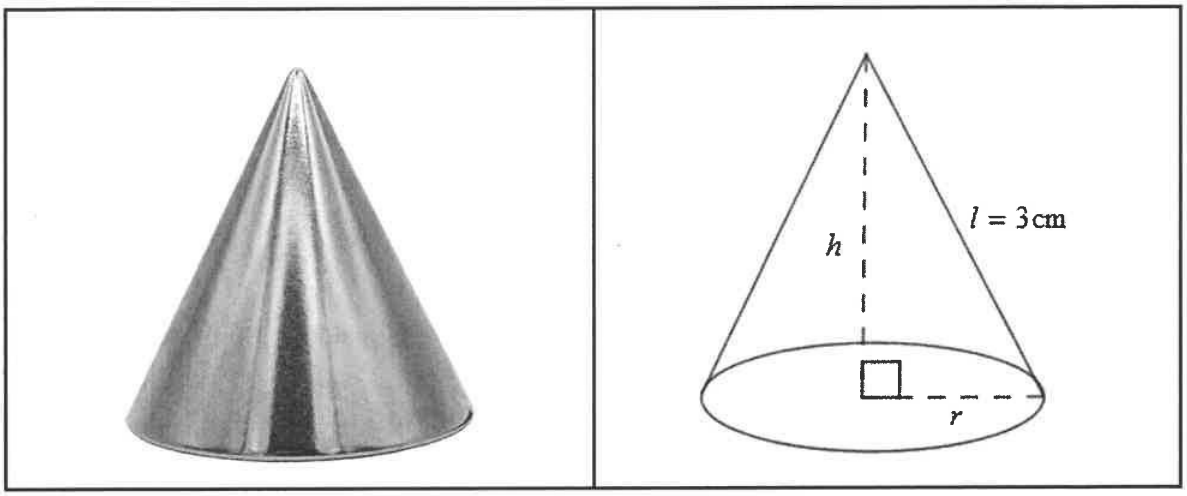
[17]

QUESTION 8

The picture and diagram below show a right cone-shaped magnet.

The right cone-shaped magnet has the following dimensions:

radius = r cm, height = h cm and slant height (l) = 3 cm



The following formula may be used:

Volume of a right cone = $\frac{1}{3}\pi r^2 \times h$

8.1 Express the radius (r) of the cone in terms of its height (h). (2)

8.2 Hence, show that the volume (V) of the cone can be expressed as $V(h) = 3\pi h - \frac{1}{3}\pi h^3$ (2)

8.3 Determine the numerical value of h for which the volume of the cone will be a maximum. (4)

[8]

QUESTION 9

9.1 Determine the following integrals:

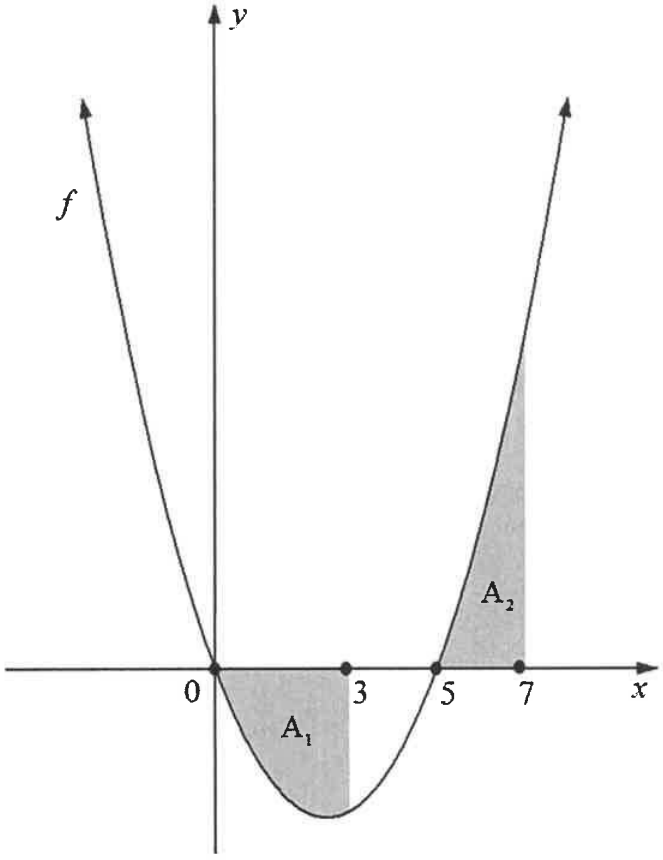
9.1.1 $\int \frac{3}{x}\,dx$ (2)

9.1.2 $\int \left(-\frac{3x}{x^5} + \sqrt[5]{x^3}\right) dx$ (4)

9.2 The sketch below shows function f defined by $f(x) = x^2 - 5x$

The two shaded areas represented are:

- A_1 = area bounded by function f, the x-axis and the ordinates $x = 0$ and $x = 3$
- A_2 = area bounded by function f, the x-axis and the ordinates $x = 5$ and $x = 7$



Determine (showing ALL calculations) by how much A_1 is greater than A_2. (9)
[15]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \; n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \; x > 0 \text{ and } k \in \mathbb{R} \; ; \; k \neq 0$$

$$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \; a > 0 \; ; \; a \neq 1 \text{ and } \; k, a \in \mathbb{R} \; ; \; k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$$

$\pi\ rad = 180°$

Angular velocity $= \omega = 2\ \pi\ n$ where n = rotation frequency

Angular velocity $= \omega = 360°\ n$ where n = rotation frequency

Circumferential velocity $= v = \pi\ D\ n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\ r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \frac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4d\,h + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = length of the equal parts, $m_1 = \frac{o_1 + o_2}{2}$
$O_n = n^{th}$ ordinate and n = number of ordinates

OR

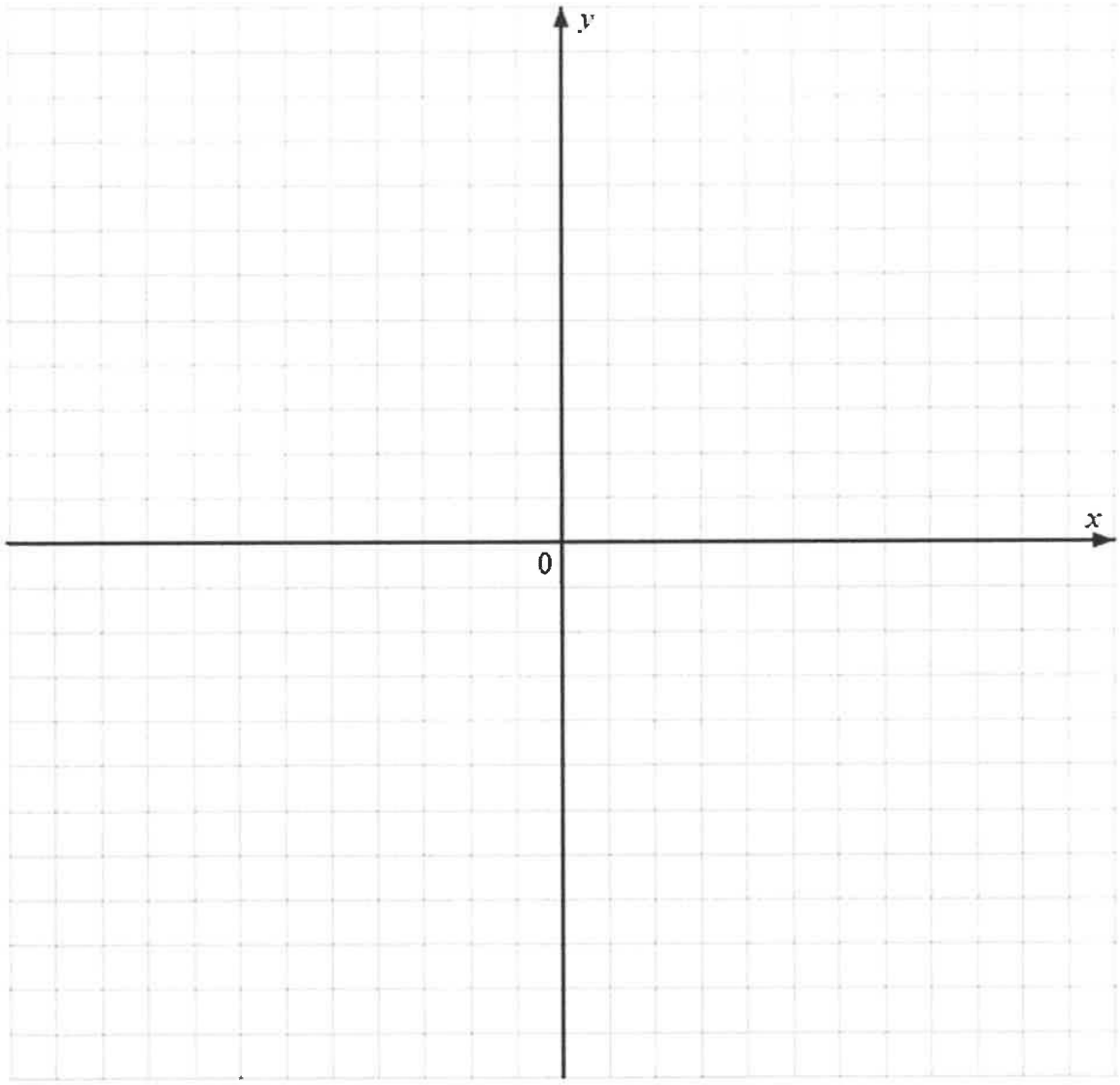
$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = length of the equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

CENTRE NUMBER							

EXAMINATION NUMBER													

QUESTION 4.3.3



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QUESTION 7.5