Province of the EASTERN CAPE EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

This question paper consists of 14 pages, including a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.

2. Answer ALL the questions.

3. Answer QUESTIONS 4.1.3 and 7.4 on the ANSWER SHEETS provided. Write your name and school's name in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.

4. Number the answers correctly according to the numbering system used in this question paper.

5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

6. Answers only will NOT necessarily be awarded full marks.

7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. If necessary, round off answers to TWO decimal places, unless stated otherwise.

9. Diagrams are NOT necessarily drawn to scale.

10. An information sheet with formulae is included at the end of the question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x+17)(x-23)=0$ (2)

1.1.2 $\frac{x^2}{2}+x-\frac{1}{3}=0$ (Correct to TWO decimal places.) (3)

1.1.3 $x(2x+1)-3\leq 0$ (4)

1.2 Solve for x and y if:

$y=x+1$ and $y=3x^2-xy$ (5)

1.3 The measure of Percentage Digestibility Coefficient (D) of a cow feed is measured as the difference between the amount of food eaten (E) and the food excreted in the faeces (F), expressed as a percentage of the food ingested.

$D=100\left(\frac{E-F}{E}\right)$; **where:**

D = Percentage Digestibility Coefficient (%)

E = Food eaten (kg)

F = Food excreted (kg)

1.3.1 Make E, the food eaten, the subject of the formula. (4)

1.3.2 Calculate the amount of food eaten by the cow if percentage digestibility coefficient is 80% and 3,75 kg of food was excreted in the faeces. (2)

1.3.3 Hence or otherwise, express, in grams, the amount of food eaten by the cow in QUESTION 1.3.2, if 1 000 g = 1 kg. (1)

1.3.4 Express the answer in QUESTION 1.3.3 in ***scientific notation***. (1)

1.4 Simplify the following binary operation, **without using a calculator**:

$1\,000_2 - 110_2$ (2)

[24]

QUESTION 2

Given: $f(x) = ax^2 - 3x + 2$

2.1 Determine the value of a if the discriminant of $f(x)$ is 6. (3)

2.2 Hence, without solving the equation, describe the nature of roots of $f(x)$. (1)

2.3 Determine the numerical value of a for which the roots of $f(x)$ are equal. (3)

[7]

QUESTION 3

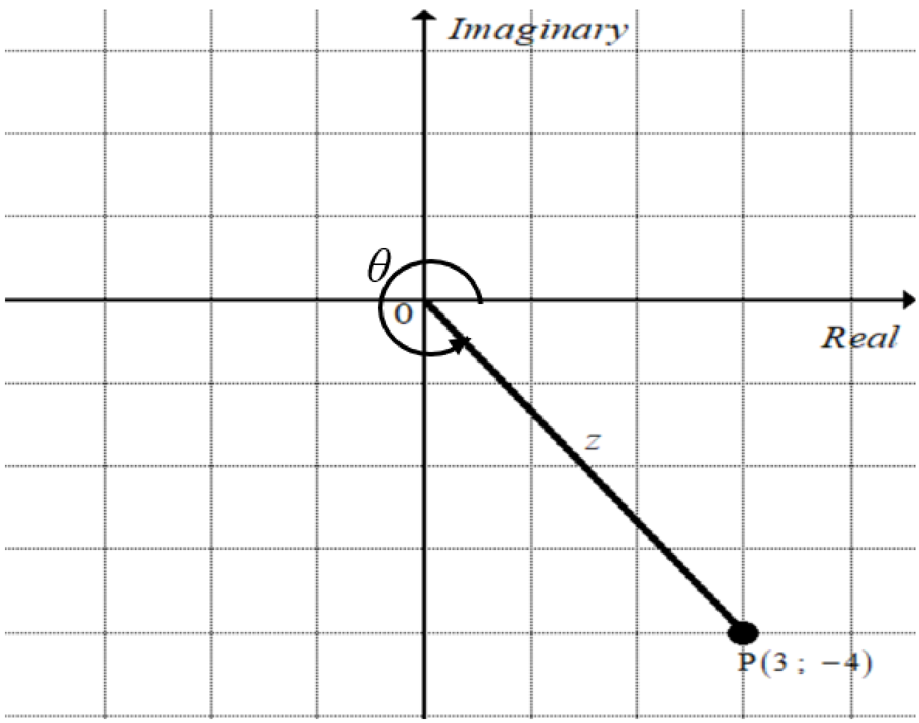
3.1 Simplify the following **without using a calculator**:

3.1.1 $\log_x\left(\frac{1}{x}\right)$ (2)

3.1.2 $4^x - 2^{2x-1}$ (3)

3.2 Show that: $\frac{\sqrt{3x^2} \times \sqrt[3]{12x^3}}{2x^2} = \frac{\sqrt[6]{243}}{\sqrt[3]{2}}$ (4)

3.3 Drawn below is an Argand diagram of complex number z with point P $(3\,;-4)$:



3.3.1 Write the complex number z in rectangular form. (1)

3.3.2 Calculate the modulus of z. (2)

3.3.3 Determine the size of θ. (3)

3.3.4 Hence express z in polar form (where θ is in degrees). (1)

3.4 Solve for x and y if: $\frac{x-i}{2i+1} = y + 3i$ (5)

[21]

QUESTION 4

4.1 Given the function *f*, defined by $f(x) = -\frac{2}{x} - 1$

4.1.1 Write down the equation of the horizontal asymptote of *f*. (1)

4.1.2 Determine the *x*-intercept of *f*. (3)

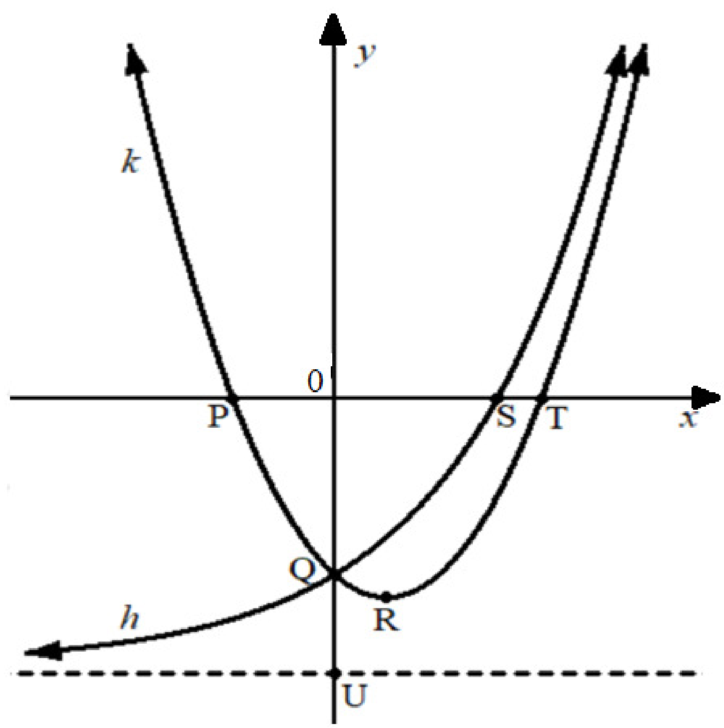
4.1.3 Hence, sketch the graph of *f* on the ANSWER SHEET provided. Clearly show the intercepts with the axes and any asymptotes. (3)

4.1.4 Write down the range of *f*. (1)

4.2 The diagram below shows sketch graphs of functions defined by:

$k(x) = x^2 - x - 2$ and $h(x) = 2^x - 3$

- Points P and T are the *x*-intercepts of *k* and S is the *x* intercept of *h*.
- Q is a common *y*-intercept for both graphs.
- R is the turning point of *k*.
- The asymptote of *h* cuts the *y*-axis at U.



Determine:

4.2.1 The *y*-coordinate of Q (1)

4.2.2 The equation of the asymptote of *h* (1)

4.2.3 The *x*-intercepts of *k* (3)

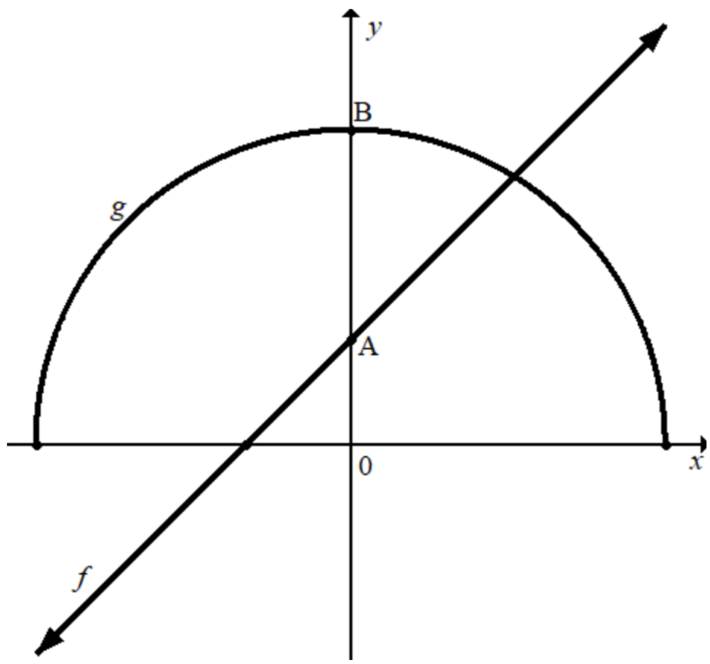
4.2.4 The coordinates of S (3)

4.2.5 The coordinates of R (4)

4.2.6 The domain of h (1)

4.2.7 The value of x for which $k(x) - h(x) = 0$ (1)

4.3 The diagram below shows sketch graphs of functions defined by:
$f(x) = x + 1$ and $g(x) = \sqrt{r^2 - x^2}$
Points A and B are the y-intercepts of f and g, respectively.



4.3.1 Determine the x-intercept of f. (2)

4.3.2 If f is shifted 2 units upwards, point **A** coincides with point **B** of function g. Hence, write down:

(a) The coordinates of point **B**. (2)

(b) The equation of a new function $h(x)$, the result of the translation of f (1)

(c) The equation of g. (1)

[28]

QUESTION 5

5.1 An asset valued at R15 000 depreciates at a rate of 3% per annum compounded quarterly. Determine the amount to which the asset depreciates at the end of 5 years. (3)

5.2 The price of brown bread increased from R3,80 in 2004 to R18,80 in 2023.

5.2.1 Determine the amount by which the price of brown bread increased from 2004 to 2023. (1)

5.2.2 Determine the inflation rate from 2004 to 2023. (5)

5.3 An artisan deposited a sum of R350 000 into an investment account that generates 7% per annum for 8 years.

- At the end of 4 years the artisan deposited an amount Rx into the investment account.
- He withdrew R100 000 at the beginning of the 6th year and invested the remaining amount at a rate of 7% per annum, compounded monthly for the remainder of the investment period.

Determine the amount, Rx, the artisan deposited at the end of the 4th year, if at the end of the 8-year investment period the investment yielded R620 000. (6)

[15]

QUESTION 6

6.1 Determine $f'(x)$ by using FIRST PRINCIPLES if $f(x) = 2 - 5x$ (5)

6.2 Determine:

6.2.1 $D_x\left(\frac{1}{\sqrt{x}} - 3kx\right)$ (4)

6.2.2 $\frac{dy}{dx}$ if: $y = \frac{2x^3 - 8x}{x - 2}$ (4)

6.3 Determine the coordinates of the point on the curve $h(x) = 3x^2 - 4x$ where the gradient of the tangent is equal to 2. (4)

[17]

QUESTION 7

Given: $f(x) = -x^3 - 4x^2 - 3x$

7.1 Determine the x-intercepts of f. (4)

7.2 Write down y-intercept of f. (1)

7.3 Determine the coordinates of the turning points of f. (5)

7.4 Sketch the graph of f on the ANSWER SHEET provided. Clearly show all the coordinates of the turning points and intercepts with the axis. (4)

7.5 Determine the average gradient of f between $x = -2$ and $x = -1$. (4)

[18]

QUESTION 8

The second Newton's Law of Motion of a body falling due to gravity is given by:

$s = ut + \frac{1}{2}gt^2$ where:

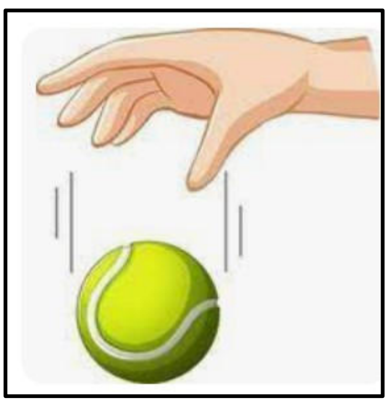
s is the displacement of an object, in meters

u is the initial velocity of an object, in m/s

t is the time taken for the fall, in seconds

g is the gravitational acceleration = 10 m/s^2

The picture below shows a ball dropped from a hand and falling to the ground.



8.1 Determine the displacement of the ball after 4 seconds if its initial velocity was 5 m/s. (2)

8.2 Write down the equation of final velocity of this object, as a function of time. (2)

8.3 Hence or otherwise, calculate the final velocity after 4 seconds. (2)

8.4 Determine the average rate of change over the period of 4 seconds. (2)

[8]

QUESTION 9

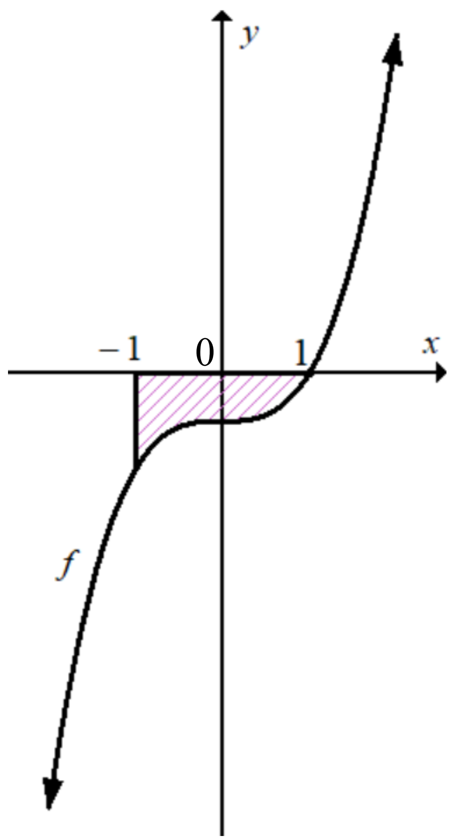
9.1 Given $f(x) = x^3 - 8$

Simplify the following integrals:

9.1.1 $\int f(x)\,dx$ (3)

9.1.2 $\int \left(\frac{f(x)}{x^2 + 2x + 4} - 2^{3x} \right) dx$ (4)

9.2 The diagram below shows the shaded area bounded by the function *f* defined by $f(x) = x^3 - 1$ and the *x*-axis between the points where $x = -1$ and $x = 1$.



Determine the area of the shaded region bounded by *f* and the *x*-axis between the point where $x = -1$ and $x = 1$ (5)

[12]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\frac{b}{2a}$ $\qquad y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 + i)^n$ $\qquad A = P(1 - i)^n$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \; n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$

$\int \frac{k}{x} dx = k \ln x + C \quad , \; x > 0 \text{ and } k \in \mathbb{R} \; ; \; k \neq 0$

$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \; a > 0 \; ; \; a \neq 1 \text{ and } k, a \in \mathbb{R} \; ; \; k \neq 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad \tan\theta = m$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$\textit{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \text{cosec}^2\theta$

$\pi \, rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° \, n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

$o_n = n^{th}$ ordinate and n = number of ordinates

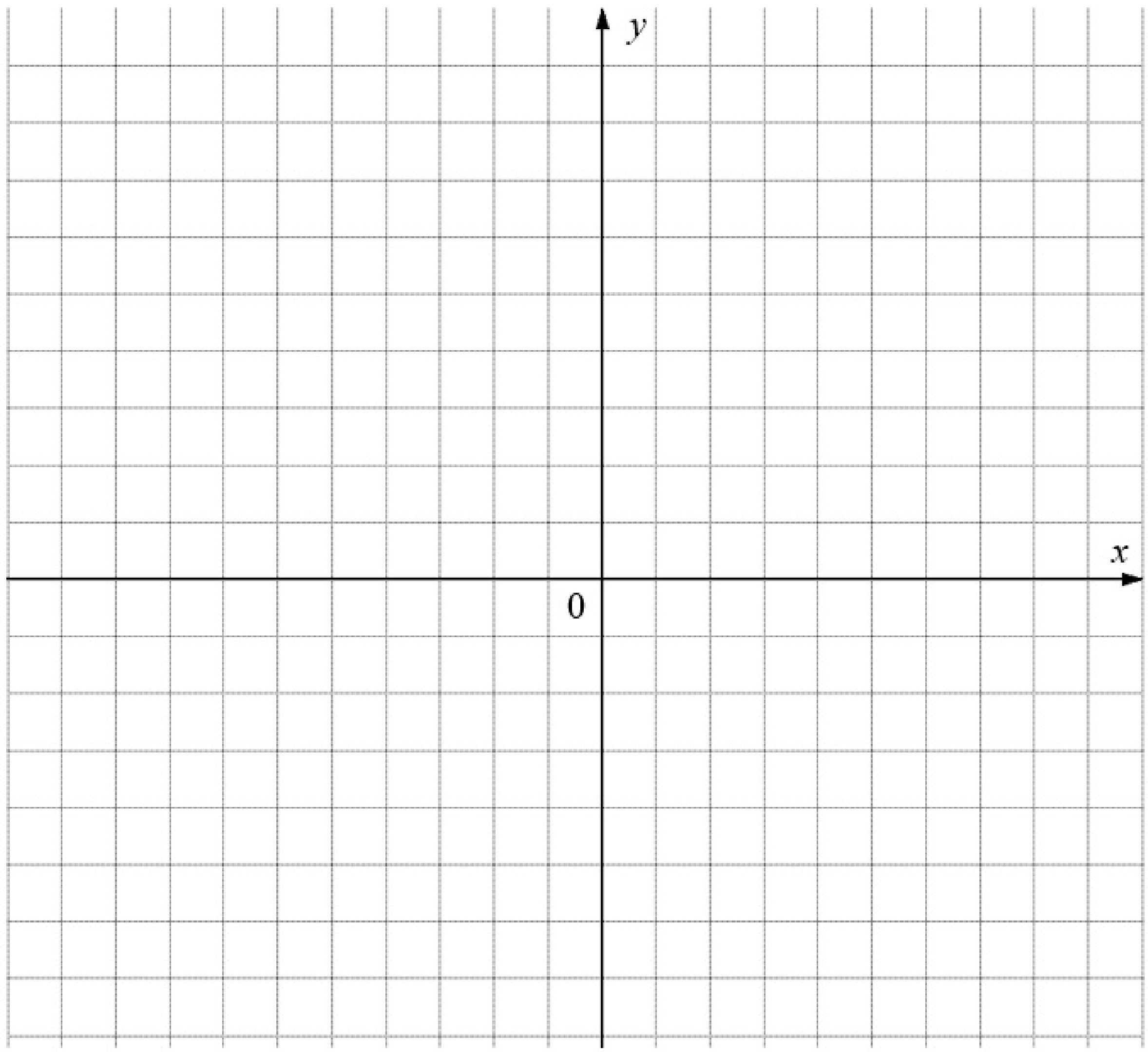
OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

DIAGRAM SHEET

SURNAME AND NAME	
SCHOOL	

QUESTION 4.1.3



QUESTION 7.4