

SA's Leading Past Year

Exam Paper Portal

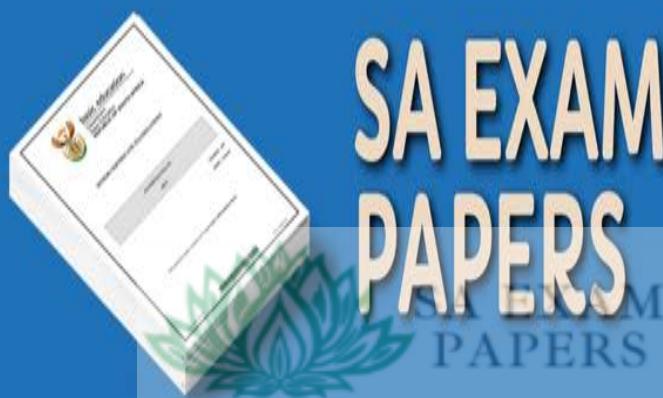
S T U D Y

You have Downloaded, yet Another Great
Resource to assist you with your Studies ☺

Thank You for Supporting SA Exam Papers

Your Leading Past Year Exam Paper Resource Portal

Visit us @ www.saexamapers.co.za





Province of the
EASTERN CAPE
EDUCATION



**NATIONAL SENIOR
CERTIFICATE/NASIONALE
SENIORSERTIFIKAAT**

GRADE/GRAAD 12

SEPTEMBER 2023

**TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

NOTE:

- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method / Metode
A	Accuracy / Akkuraatheid
AO	Answer only / Slegs antwoord
CA	Consistent accuracy / Deurlopende akkuraatheid
F	Formula / Formule
I	Identity / Identiteit
R	Rounding / Afronding
S	Simplification / Vereenvoudiging
ST	Statement / Bewering
RE	Reason / Rede
ST RE	Statement and correct reason / Bewering en korrekte rede
SF	Substitution correctly in correct formula / Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units / Geen penalisering vir eenhede uitgelaat

QUESTION/VRAAG 1

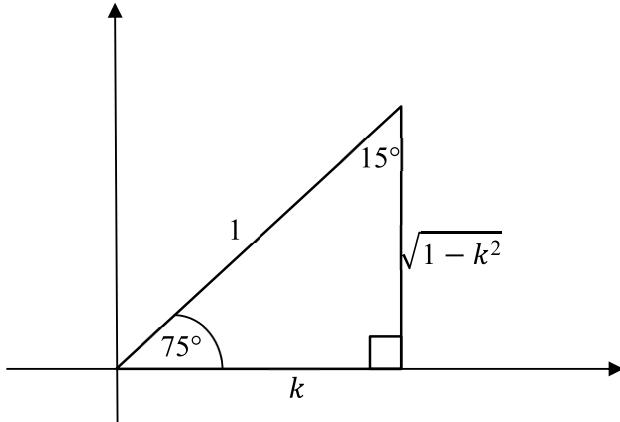
1.1	$m_{AD} = \frac{6-4}{3-(-1)}$ $m_{AD} = \frac{1}{2}$ <p style="text-align: right;">AO: Full marks / Volpunte</p>	✓ SF A ✓ $\frac{1}{2}$ CA (2)
1.2	$P\left(\frac{3+4}{2}; \frac{6+1}{2}\right)$ $P\left(\frac{7}{2}; \frac{7}{2}\right)$	✓ SF A ✓ $\left(\frac{7}{2}; \frac{7}{2}\right)$ CA (2)
1.3	$m_{AB} = m_{CD} = \frac{1}{2}$ (lines / lyne) $\therefore y = \frac{1}{2}x + c$ $(4; 1): 1 = \frac{1}{2}(4) + c$ $\therefore 1 = 2 + c$ $\therefore -1 = c$ $\therefore y = \frac{1}{2}x - 1$ <div style="text-align: center; margin-top: 20px;"> OR </div> $\therefore y - y_1 = \frac{1}{2}(x - x_1)$ $(4; 1): y - 1 = \frac{1}{2}(x - 4)$ $\therefore y - 1 = \frac{1}{2}x - 2$ $\therefore y = \frac{1}{2}x - 2 + 1$ $\therefore y = \frac{1}{2}x - 1$	✓ $m = \frac{1}{2}$ CA ✓ sub/verv (4; 1) CA ✓ $y = \frac{1}{2}x - 1$ A (3)
1.4	$0 = \frac{1}{2}x - 1$ $1 = \frac{1}{2}x$ $2 = x$ $\therefore E(2; 0)$	✓ $y = 0$ A ✓ (2; 0) CA (2)

1.5	$m_{AE} = \frac{4-0}{-1-2}$ $m_{AE} = \frac{4}{-3}$ $\tan\theta = m_{AE}$ $\tan\theta = \frac{4}{-3}$ $\therefore \text{ref.}\angle = \tan^{-1}\left(\frac{4}{3}\right) = 53,13^\circ$ $\therefore \theta = 180^\circ - 53,13^\circ$ $\therefore \theta = 126,87^\circ$	✓ $m_{AE} = \frac{4}{-3}$ CA ✓ SF CA ✓ ref.∠ CA ✓ $\theta = 126,87^\circ$ CA (4)
1.6	$m_{CE} = \frac{1}{2}$ $\tan\alpha = m_{CE}$ $\tan\alpha = \frac{1}{2}$ $\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26,57^\circ$ $\therefore A\hat{E}D = 126,87^\circ - 26,57^\circ = 100,30^\circ$	✓ $m_{CE} = \frac{1}{2}$ A ✓ SF CA ✓ $\alpha = 26,57^\circ$ CA ✓ $A\hat{E}D = 100,30^\circ$ CA (4)
		[17]

QUESTION/VRAAG 2

2.1		
2.1.1	$m_{OQ} = \frac{3-0}{2-0}$ $m_{OQ} = \frac{3}{2}$ <p style="text-align: right;">AO: Full marks / Volpunte</p>	✓ SF ✓ m_{OQ} A CA (2)
2.1.2	$m_{tan} = -\frac{2}{3}$ $\therefore y = -\frac{2}{3}x + c$ $(2; 3): 3 = -\frac{2}{3}(2) + c$ $\therefore 3 = -\frac{4}{3} + c$ $\therefore \frac{13}{3} = c$ $\therefore y = -\frac{2}{3}x + \frac{13}{3}$ <p style="text-align: center;">OR/OF</p> $xx_1 + yy_1 = r^2$ $\therefore x(2) + y(3) = 13$ $\therefore y = -\frac{2}{3}x + \frac{13}{3}$	✓ $m = -\frac{2}{3}$ CA ✓ sub/verv (2 ; 3) CA ✓ equation/vergelyking CA OR/OF ✓ F ✓ SF ✓ equation/vergelyking CA (3)
2.2		✓ elliptical shape with horizontal axis as major axis / Elliptiese vorm met groter-as as die horizontale-as ✓ x-intercepts/ x-afsnitte ✓ y-intercepts/ y-afsnitte (3)
		[8]

QUESTION/VRAAG 3

3.1.1	$\cos(128,2^\circ + 204,7^\circ) \approx 0,89$ AO: Full marks / Volpunte	✓ SF ✓ 0,89 (2)	A A
3.1.2	$\operatorname{cosec}(204,7^\circ - 128,2^\circ)$ $= \frac{1}{\sin(76,5^\circ)}$ $\approx 1,03$ AO: Full marks / Volpunte	✓ SF ✓ reciprocal ratio / resiprook verh. ✓ 1,03 (3)	A A A
3.2.1	 $\therefore \sin 15^\circ = k$	✓ Diagram ✓ $\sqrt{1 - k^2}$ ✓ k (3)	A A CA (3)
3.2.2	$\tan 255^\circ = \tan(180^\circ + 75^\circ)$ $\therefore \tan 75^\circ = \frac{\sqrt{1-k^2}}{k}$	✓ Reduction/reduksie ✓ numerator/teller CA ✓ denominator/noemer CA (3)	A CA CA (3)
3.3	$\sec \theta = -1,583$ $\frac{1}{\cos \theta} = -1,583$ $1 = -1,583 \times \cos \theta$ $\frac{1}{-1,583} = \cos \theta$ $50,82 \dots^\circ = \text{ref. } \angle$ $\therefore \text{Q2: } \theta = 180^\circ - 50,82^\circ = 129,18^\circ$ AND/EN $\therefore \text{Q3: } \theta = 180^\circ + 50,82^\circ = 230,82^\circ$	✓ $\frac{1}{-1,583} = \cos \theta$ ✓ ref. \angle ✓ Quadrants/kwadrante ✓ Both answers/beide antwoorde (4)	A A A CA (4)
		[15]	

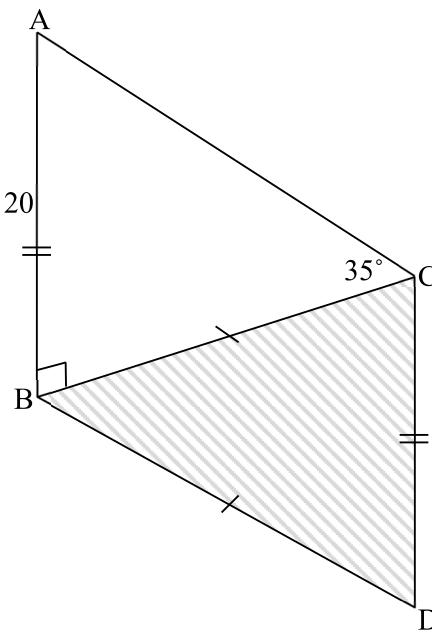
QUESTION/VRAAG 4

4.1	$\begin{aligned} & \operatorname{cosec}^2(180^\circ + \theta) + \frac{\sin(180^\circ - \theta) \cdot \cot^2(180^\circ + \theta) \cdot \sin 270^\circ}{\cos(360^\circ - \theta) \cdot \tan(180^\circ + \theta)} \\ &= \operatorname{cosec}^2 \theta + \frac{(\sin \theta) \cdot (\cot^2 \theta) \cdot (-1)}{(\cos \theta) \cdot (\tan \theta)} \\ &= \operatorname{cosec}^2 \theta - \frac{\tan \theta \cdot \cot^2 \theta}{\tan \theta} \\ &= \operatorname{cosec}^2 \theta - \cot^2 \theta \\ &= 1 \end{aligned}$	✓ cosec ² θ A ✓ sinθ A ✓ cot ² θ A ✓ -1 A ✓ cosθ A ✓ tanθ A ✓ tan θ I A ✓ S A ✓ 1 I A (9)
4.2	$\begin{aligned} & \frac{1}{(1 - \sin \theta)(1 + \sin \theta)} = \sec^2 \theta \\ \therefore \text{LHS} &= \frac{1}{(1 - \sin \theta)(1 + \sin \theta)} \\ \therefore \text{LHS} &= \frac{1}{1 - \sin^2 \theta} \\ \therefore \text{LHS} &= \frac{1}{\cos^2 \theta} \\ \therefore \text{LHS} &= \sec^2 \theta = \text{RHS} \end{aligned}$	✓ 1 - sin ² θ A ✓ $\frac{1}{\cos^2 \theta}$ A (2)
		[11]

QUESTION/VRAAG 5

5.1		
	<p>✓ f and g start points and end points / beginpunte en eindpunte A</p> <p>g:</p> <ul style="list-style-type: none"> ✓ shape/vorm A ✓ turning points/draaipunte A ✓ y-intercept/y-afsnit A <p>f:</p> <ul style="list-style-type: none"> ✓ shape/vorm A ✓ x-intercepts/x-afsnitte A ✓ Asymptotes/Asimptote A 	(7)
5.2	$y \in [-2 ; 0]$ OR/OF $-2 \leq y \leq 0$	<ul style="list-style-type: none"> ✓ Notation/notasie A ✓ start and endpoints/begin- en eindpunte CA (2)
5.3	180°	<ul style="list-style-type: none"> ✓ Answer/Antw. A (1)
5.4	$x \in (180^\circ; 270^\circ)$ OR/OF $180^\circ < x < 270^\circ$	<ul style="list-style-type: none"> ✓ endpoints / eindpunte CA ✓ notation /notasie CA (2)
		[12]

QUESTION/VRAAG 6



6.1	$\tan 35^\circ = \frac{20}{BC}$ $BC \times \tan 35^\circ = 20$ $BC = \frac{20}{\tan 35^\circ}$ $BC = 28,56 \text{ units/eenhede}$	✓ SF A ✓ BC Subj /onderwerp CA ✓ answer/antw CA (3)
6.2	$a^2 = b^2 + c^2 - 2b.c.\cos A$ $CD^2 = BC^2 + BD^2 - 2BC.BD.\cos C\widehat{B}D$ $(20)^2 = (28,56)^2 + (28,56)^2 - 2(28,56)(28,56)\cos C\widehat{B}D$ $0,75 \dots = \cos C\widehat{B}D$ $40,99 \dots^\circ = C\widehat{B}D$ $\therefore C\widehat{B}D = 41^\circ$	✓ F A ✓ SF CA ✓ S CA ✓ Answer/Antw. A (4)
6.3	$\text{Area } \Delta ABCD = \frac{1}{2} \cdot a \cdot b \cdot \sin C$ $\text{Area } \Delta ABCD = \frac{1}{2} \cdot BC \cdot BD \cdot \sin C\widehat{B}D$ $\text{Area } \Delta ABCD = \frac{1}{2} (28,56)(28,56) \sin 41^\circ$ $\text{Area } \Delta ABCD = 267,57 \text{ square units}$	✓ F A ✓ SF CA ✓ Answer/Antw. A (3)
		[10]

QUESTION/VRAAG 7

7.1	Equal/gelyk	✓ (1)
7.2		
7.2.1	$\hat{B}_3 = 80^\circ$ (ext. \angle of cq / buite \angle van kvh) $\hat{C}_1 = 80^\circ$ (\angle 's opp = sides / \angle 'e teenoor = sye) $\hat{B}_1 = 80^\circ$ (\angle 's in same seg. / \angle 'e in dieselfde seg.) 	✓ ST ✓RE ✓ ST ✓RE ✓ ST ✓RE (6)
7.2.2	(a) $\hat{C}_3 = 70^\circ$ (ext. \angle of cq / buite \angle van kvh)	✓ ST ✓RE (2)
	(b) $\hat{E} = 30^\circ$ (int. \angle of Δ / binne \angle van Δ)	✓ ST ✓RE (2)
	(c) $\hat{D}_1 = \hat{C}_2$ (\angle 's in same seg. / \angle 'e in dieselfde seg.) $\therefore \hat{C}_2 = 180^\circ - 70^\circ - 80^\circ$ (\angle 's on a straight line / \angle 'e op 'n reguitlyn) $\therefore \hat{D}_1 = 30^\circ$	✓ ST ✓RE ✓ ST OR/OF ✓ ST ✓RE ✓ ST (3)
7.2.3	$\hat{D}_1 = \hat{E} = 30^\circ$ \therefore AD is a tangent / AD is 'n raaklyn (\angle between the line and a chord / \angle tussen die lyn en 'n koord)	✓ ST ✓ RE (2)
		[16]

QUESTION/VRAAG 8

8.1	twice / twee keer	✓	(1)
8.2			
8.2.1	$\hat{L}_1 = 76^\circ$ (\angle 's in same seg. / \angle 'e in dieselfde seg.)	✓ ST ✓RE	(2)
8.2.2	$\hat{O}_1 = 76^\circ$ (corresp. \angle 's =; ON LM / ooreenk. \angle 'e =; ON LM)	✓ ST ✓RE	(2)
8.2.3	$\hat{M}_4 = 38^\circ$ (\angle at centre = $2 \times \angle$ at circumf. / middelpnts \angle = $2 \times$ omtreks \angle)	✓ ST ✓RE	(2)
8.2.4	$\hat{N}_1 + \hat{N}_2 = 104^\circ$ (opp \angle 's of cq / teenoorst. \angle 'e van kvh)	✓ ST ✓RE	(2)
			[9]

QUESTION/VRAAG 9

9.1	Divides / verdeel	✓ A (1)
9.2		
9.2.1	<p>In ΔBDC:</p> $\widehat{D} = 90^\circ \text{ (\angle in semi-circle / \angle in semi-sirkel)}$ $\therefore DC^2 = 17^2 - 8^2 \text{ (Pyth)}$ $\therefore DC = 15 \text{ cm}$	✓ ST ✓ RE ✓ SF ✓ Answer / Antw. CA (4)
9.2.2	<p>(a) $\frac{CE}{BC} = \frac{CF}{CD}$ (line \parallel one side of Δ / lyn \parallel een sy van Δ)</p> $\frac{1}{4} = \frac{CF}{15}$ $\therefore CF = 3,75 \text{ cm}$	✓ ST ✓ RE ✓ SF ✓ Answer / Antw. CA (4)
	<p>(b) In ΔBAC and/en ΔFEC:</p> <ol style="list-style-type: none"> 1. \hat{C} is a common angle/gemene hoek 2. $E\hat{B}A = 90^\circ$ (rad \perp tan) $E\hat{F}C = 90^\circ$ (corresp. \angle's =; $BD \parallel EF$ / ooreenk. \angle'e =; $BD \parallel EF$) $\therefore E\hat{B}A = E\hat{F}C$ 3. $C\hat{E}F = B\hat{A}C$ (int. \angle of Δ / binne \angle van Δ) $\therefore \Delta BAC \parallel \Delta FEC$ <p style="text-align: center;">OR / OF</p> <p>In ΔBAC and/en ΔFEC:</p> <ol style="list-style-type: none"> 1. \hat{C} is a common angle/gemene hoek 2. $E\hat{B}A = 90^\circ$ (rad \perp tan) $E\hat{F}C = 90^\circ$ (corresp. \angle's =; $BD \parallel EF$ / ooreenk. \angle'e =; $BD \parallel EF$) $\therefore E\hat{B}A = E\hat{F}C$ <p>$\therefore \Delta BAC \parallel \Delta FEC$ ($\angle\angle\angle$)</p>	✓ ST ✓ ST ✓ RE ✓ ST RE ✓ ST OR / OF ✓ ST ✓ ST ✓ RE ✓ ST RE ✓ RE (5)

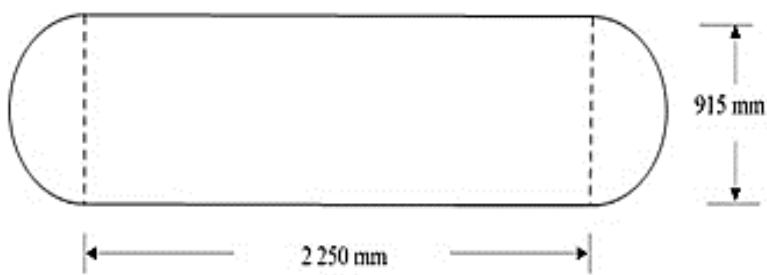
	(c)	<p>In ΔBAD and/<i>en</i> ΔCBD:</p> <ol style="list-style-type: none"> 1. $B\hat{D}A = B\hat{D}C$ (\angle's on str. line/\angle 'e op reguit lyn) 2. $A\hat{B}D = B\hat{C}D$ (tanchord/raaklynkoord) 3. $B\hat{A}D = D\hat{B}C$ (int. \angle of Δ / binne \angle van Δ) <p>$\therefore \Delta BAD \parallel\parallel \Delta CBD$ ($\angle\angle\angle$)</p> $\therefore \frac{BA}{CB} = \frac{AD}{BD} = \frac{BD}{CD} (\Delta BAD \parallel\parallel \Delta CBD)$ $\therefore \frac{AD}{8} = \frac{15}{15}$ $\therefore AD = \frac{64}{15} \approx 4,27 \text{ cm}$ <p style="text-align: center;">OR/OF</p> $\frac{AC}{EC} = \frac{BC}{FC} (\Delta BAC \parallel\parallel \Delta FEC)$ <p>But, maar $EC = \frac{1}{4} BC = \frac{17}{4}$ and/<i>en</i></p> $FC = \frac{1}{4} DC = \frac{15}{4} (\text{Prop th/ewerig st. } BD \parallel EF)$ $\therefore \frac{AC}{\frac{17}{4}} = \frac{17}{\frac{15}{4}}$ $\therefore AC \approx 19,27$ $\therefore AD = 19,27 - 15 = 4,27$	✓ ST RE ✓ ST RE ✓ ST ✓ Answer / Antw. CA
			(4) [18]

QUESTION/VRAAG 10

10.1.1	$n = \frac{\text{No. of rev.}}{\text{time}} = \frac{1 \text{ rev}}{50 \text{ min}} = 0,02 \text{ rev/min}$			✓ conv. /Herleid A (1)
10.1.2	$14\ 6425 \text{ km} = 14,6425 \times 1\ 000 = 14\ 642,5 \text{ m}$			✓ conv. /Herleid A (1)
10.1.3	$V = \pi Dn$ $V = \pi(14\ 642,5)(0,02)$ $V \approx 920,02 \text{ m/min}$			✓ F A ✓ SF CA ✓ Answer/Antw.CA (3)
10.2	$n = 15 \text{ rev/sec} = 900 \text{ rev/min}$ $\omega = 2\pi n$ $\omega = 2\pi(900)$ $\omega \approx 5\ 654,87 \text{ rad/min}$			✓ conv. /Herleid A ✓ F A ✓ SF CA ✓ Answer/Antw.CA (4)
10.3	10.3.1	$80 \text{ mm} = 8 \text{ cm}$ $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $\div 4: h^2 - 10h + 16 = 0$ $(h - 8)(h - 2) = 0$ $\therefore h = 8 \text{ or } h = 2$ $\therefore h = 2 \text{ cm}$	$80 \text{ mm} = 8 \text{ cm}$ $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $\div 4: h^2 - 10h + 16 = 0$ $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $h = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(16)}}{2(1)}$ $\therefore h = 8 \text{ or } h = 2$ $\therefore h = 2 \text{ cm}$	✓ F A ✓ SF CA ✓ Factors/faktore A ✓ Answer/Antw.CA ✓ $h = 2 \text{ cm}$ A (5)
	10.3.2	$r = 10 \div 2 = 5$ $s = r\theta$ $5,24 = 5\theta$ $1,048 \text{ rad} = \theta$ $\therefore \theta = 1,048 \times \frac{180^\circ}{\pi}$ $\therefore \theta = 60^\circ$		✓ F A ✓ SF CA ✓ conv. /Herleid A ✓ Answer/Antw. CA (4)
	10.3.3	$\text{Area} = \frac{r^2\theta}{2}$ $\text{Area} = \frac{(5)^2(1,048)}{2}$ $\text{Area} = 13,1 \text{ cm}^2$	$\text{Area} = \frac{rs}{2}$ $\text{Area} = \frac{(5)(5,25)}{2}$ $\text{Area} = 13,1 \text{ cm}^2$	✓ F A ✓ SF CA ✓ Area CA (3)
				[21]

QUESTION/VRAAG 11

<p>11.1</p>	<p>$\text{Area} = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \dots + o_{n-1} \right)$</p> $149,38 = \frac{x}{5} \left(\frac{12+12,5}{2} + 12,3 + 11,8 + 11,6 + 11,8 \right)$ $149,38 = \frac{x}{5} (59,75)$ $\frac{x}{5} = \frac{149,38}{59,75} \approx 2,50$ $x = 12,5 \text{ cm}$ <p>OR/OF</p> $\text{Area} = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$ $149,38 = \frac{x}{5} \left(\frac{12+12,3}{2} + \frac{12,3+11,8}{2} + \frac{11,8+11,6}{2} + \frac{11,6+11,8}{2} + \frac{11,8+12,5}{2} \right)$ $149,38 = \frac{x}{5} (12,15 + 12,05 + 11,7 + 11,7 + 12,15)$ $149,38 = \frac{x}{5} (59,75)$ $\frac{x}{5} = \frac{149,38}{59,75} \approx 2,50$ $x = 12,5 \text{ cm}$	<p>✓ F A ✓ $a = \frac{x}{5}$ A ✓ SF CA ✓ value of / waarde van x CA</p> <p>(4)</p> <p>OR / OF</p> <p>✓ F A ✓ $a = \frac{x}{5}$ A ✓ SF CA ✓ value of / waarde van x CA</p> <p>(4)</p>
-------------	--	---

11.2		
11.2.1	$915 \text{ mm} = 91,5 \text{ cm}$ and/en $2250 \text{ mm} = 225 \text{ cm}$	✓ conversion /herleiding A (1)
11.2.2	<p>Volume of storage tank = Volume of sphere + volume of cylinder</p> $\begin{aligned} \text{Volume van opgaartenk} &= \frac{4}{3}\pi r^3 + \pi r^2 h \\ &= \frac{4}{3}\pi \left(\frac{91,5}{2}\right)^3 + \pi \left(\frac{91,5}{2}\right)^2 \times 225 \\ &= 598\,615,875\pi \text{ cm}^3 \\ &= 598,62 \text{ litres} \end{aligned}$	✓ radius A ✓ F A ✓ SF A ✓ ans in litres / antw in liters CA (4)
11.2.3	<p>$0,5 \text{ ton} = 500 \text{ kg}$</p> <p>$\therefore \text{Weight/Gewig of/van gas} = 598,62 \text{ l} \times \frac{1 \text{ kg}}{1,96 \text{ l}} = 305,42 \text{ kg}$</p> <p>$\therefore \text{Tare weight/tarra gewig} = 500 - 305,42 = 194,58 \text{ kg}$</p> <p>$\therefore \text{Tare weight \% / tarra gewig \%} = \frac{194,58}{500} \times 100\% \approx 38,92\%$</p>	✓ conversion / herleiding A ✓ Weight of gas / gewig van gas CA ✓ tare weight / tarra gewig CA ✓ tare perct /tarra perst CA (4) [13]
TOTAL/TOTAAL: 150		