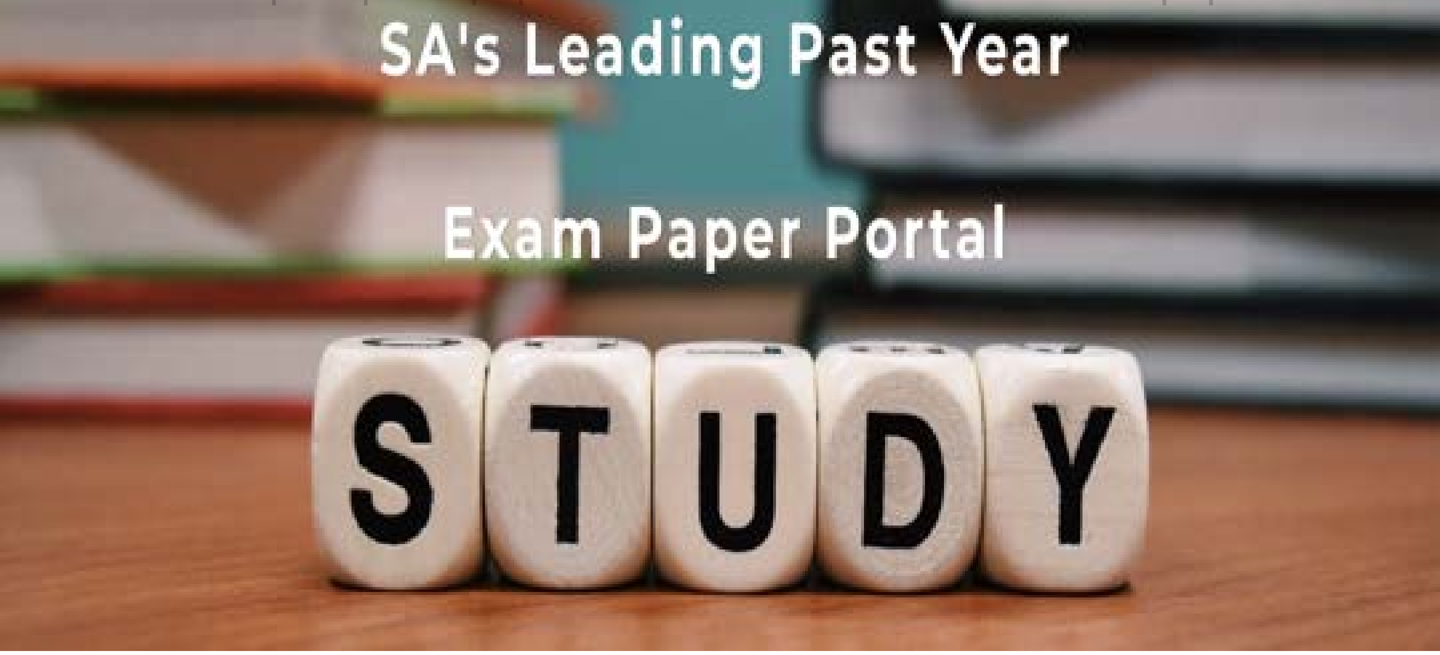
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STUDY

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2023

10612

MATHEMATICS

(PAPER 2)

TIME: 3 hours

MARKS: 150

14 pages + 1 information sheet and a 27-page answer book

MATHEMATICS: Paper 2
10612E

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

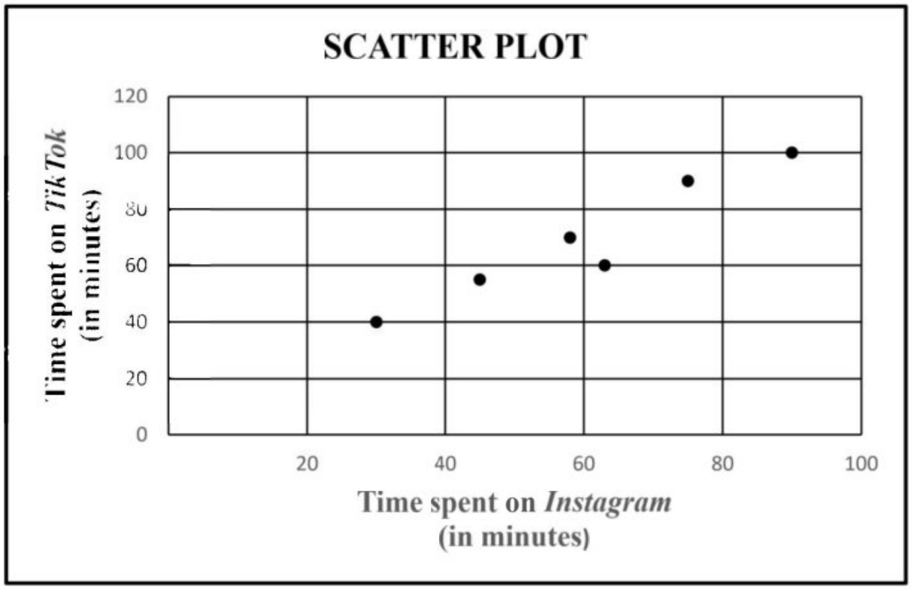
1. This question paper consists of TEN questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A survey was conducted among a group of learners to compare the time spent on *Instagram* to the time spent on *TikTok*.

The results are shown in the table below.

TIME SPENT ON *INSTAGRAM* (in minutes)	30	45	58	63	75	90
TIME SPENT ON *TIKTOK* (in minutes)	40	55	70	60	90	100



1.1 Calculate the correlation coefficient of the data. (1)

1.2 Comment on the strength of the correlation between the time spent on *Instagram* and the time spent on *TikTok*. (1)

1.3 Determine the equation of the least squares regression line for the data. (3)

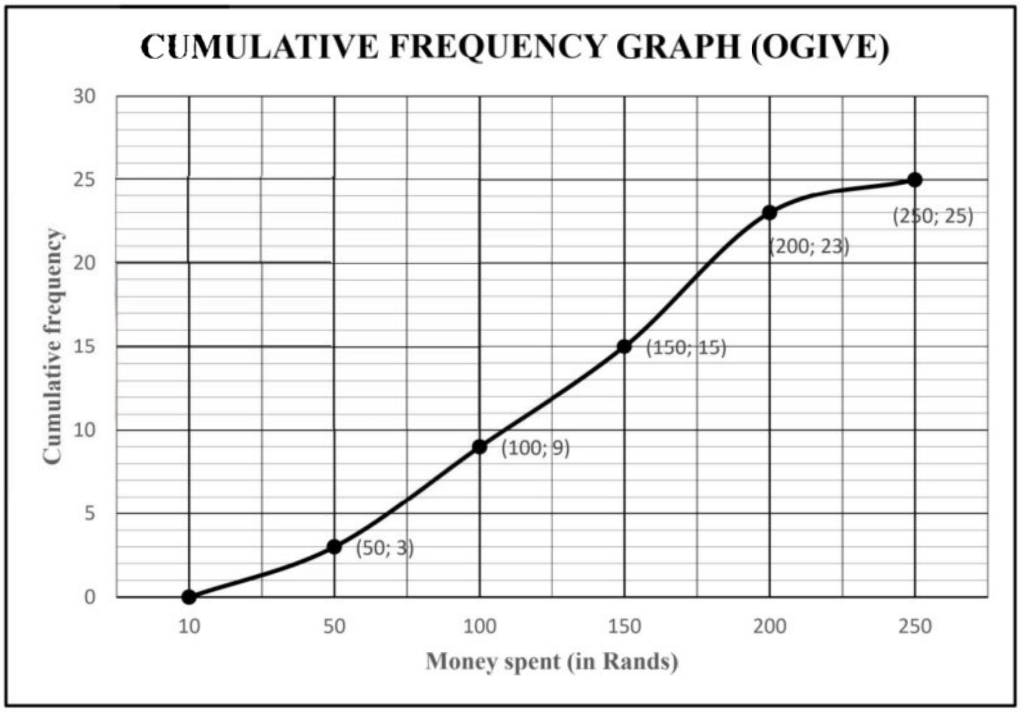
1.4 Predict the time that will be spent on *TikTok* if a learner spent 115 minutes on *Instagram*. (2)

1.5 It was noticed that 4 learners' data was not recorded. The mean time of the *TikTok* users and *Instagram* users was 73,4 minutes each. The researcher commented that the total amount of time spent on the two social media platforms was more than a full day. Do you agree with the researcher?

Motivate your answer by using necessary calculations. (3)
[10]

QUESTION 2

The amount of money (in rands) that a group of learners spent at a theme park on a specific day was recorded. The data is represented in the cumulative frequency graph (ogive) below.



2.1 The data from the cumulative frequency graph (ogive) is represented in the incomplete frequency table below.

AMOUNT OF MONEY (IN RANDS)	NUMBER OF LEARNERS
$10 \leq x < 50$	a
$50 \leq x < 100$	6
$100 \leq x < 150$	b
$150 \leq x < 200$	8
$200 \leq x < 250$	2

2.1.1 How many learners visited the theme park on that specific day? (1)

2.1.2 Determine the values of ***a*** and ***b*** in the frequency table. (2)

2.1.3 Use the ogive to determine the percentage of learners that spent more than R175. (2)

2.2 It is further given that there are two rides at the theme park, *The Intimidator* and *Terror Thrills*.

The mean amount of money spent on these rides was analysed and is given below.

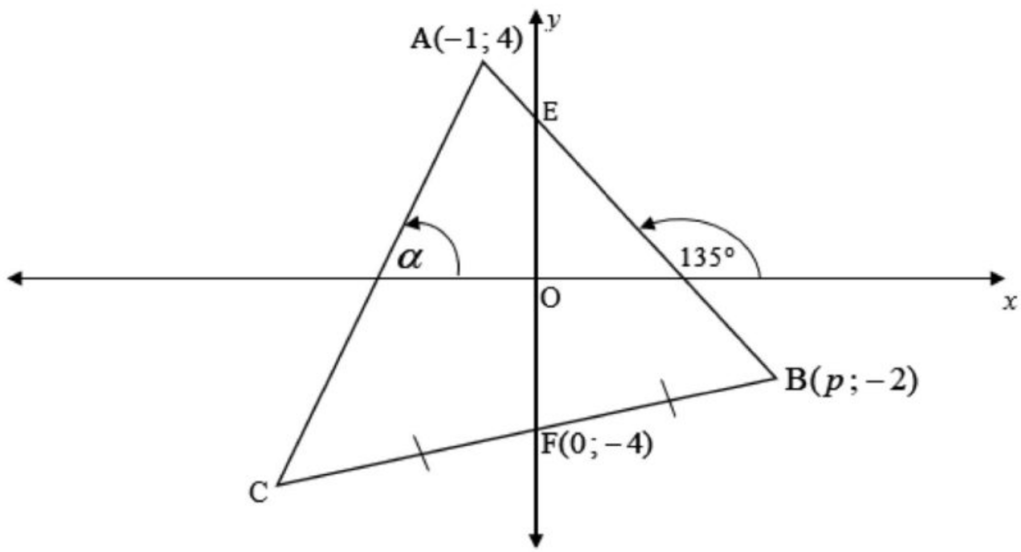
Rides	***The Intimidator***	***Terror Thrills***
Mean amount spent	R13,20	R12,70

The two standard deviations interval about the mean for *The Intimidator* was calculated as (4,8 ; 9,2). If the standard deviation of *Terror Thrills* is double the standard deviation of *The Intimidator*, calculate the interval for the one standard deviation about the mean for *Terror Thrills*. (4)

[9]

QUESTION 3

In the diagram below, A$(-1\,;4)$, B$(p\,;-2)$ and C, are vertices of ΔABC. E is the y-intercept of AB. F$(0\,;-4)$ is the midpoint of BC. The angles of inclination of AB and AC are 135° and α respectively.



3.1 Calculate the gradient of AB. (2)

3.2 Show that the value of p is 5. (2)

3.3 Calculate the coordinates of C. (2)

3.4 Determine the equation of AC in the form $y = mx + c$. (4)

3.5 Calculate the size of $C\hat{A}B$. (3)

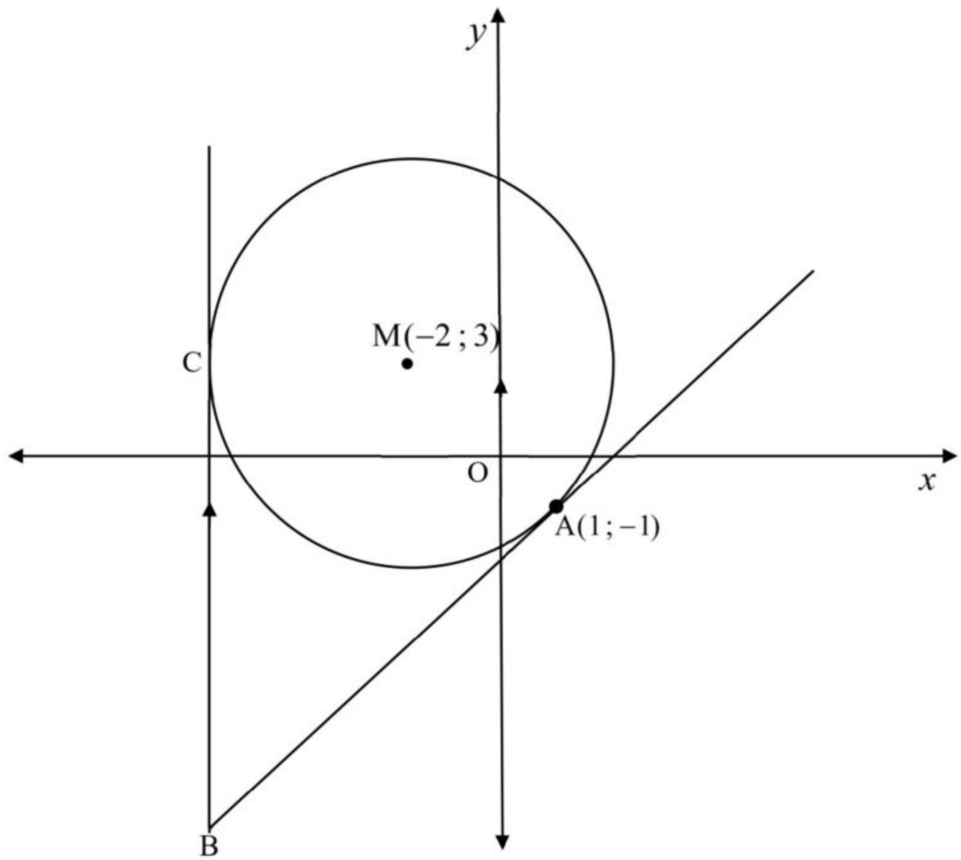
3.6 Calculate the area of ΔBEF. (3)

3.7 Another point K$(t\,;t)$ where $t < 0$, is plotted such that AK $= 5\sqrt{5}$.
Calculate the coordinates of K. (5)

[21]

QUESTION 4

In the diagram below, the circle centred at M(–2 ; 3) passes through A(1 ; –1) and C. BA and BC are tangents to the circle at A and C respectively, with BC parallel to the y-axis.



4.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.2 Write down the coordinates of C. (2)

4.3 Determine the equation of the tangent AB in the form $y=mx+c$. (5)

4.4 Determine the length of BC. (3)

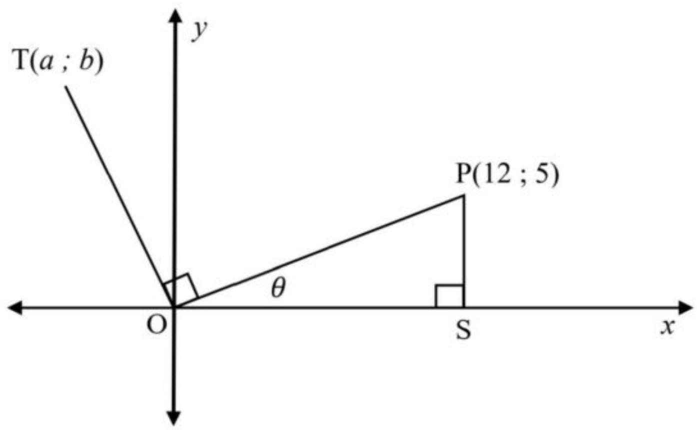
4.5 Determine the equation of the circle centred at A that has both the x- and y-axis as tangents. (2)

4.6 If another circle with centre N(p ; 3) and a radius of 4 intersects the circle centred at M at two distinct points, determine all the possible values of p. (5)

[20]

QUESTION 5

5.1 In the diagram below, P is the point (12 ; 5) and T(a ; b). $OT \perp OP$; $PS \perp x$-axis and $P\hat{O}S = \theta$.



Without using a calculator, determine, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin\theta$ (2)

5.1.3 a, if TO = 19,5 units (4)

5.2 Determine the value of the following, **without using a calculator**:

$$\frac{\sin(360° - 2x).\sin(-x)}{\sin(90° + x)} + 2\cos^2(180° + x)$$ (6)

5.3 Given: $\cos 42° = \sqrt{k}$

Without using a calculator, determine the value of $\sin^2 69°$ in terms of k. (3)

5.4 Given the identity: $\frac{\sin 5x.\cos 3x - \cos 5x.\sin 3x}{\tan 2x} - 1 = -2\sin^2 x$

5.4.1 Prove the identity. (4)

5.4.2 Determine the values of x for which the identity will be undefined in the interval $x \in [0° ; 60°]$. (2)

5.5 Given: $f(x) = 2\cos x - \sin^2 x$

5.5.1 Show that $f(x)$ can be expressed as $f(x) = (\cos x + 1)^2 - 2$. (2)

5.5.2 Hence, or otherwise, find the maximum value of f. (2)

[26]

QUESTION 6

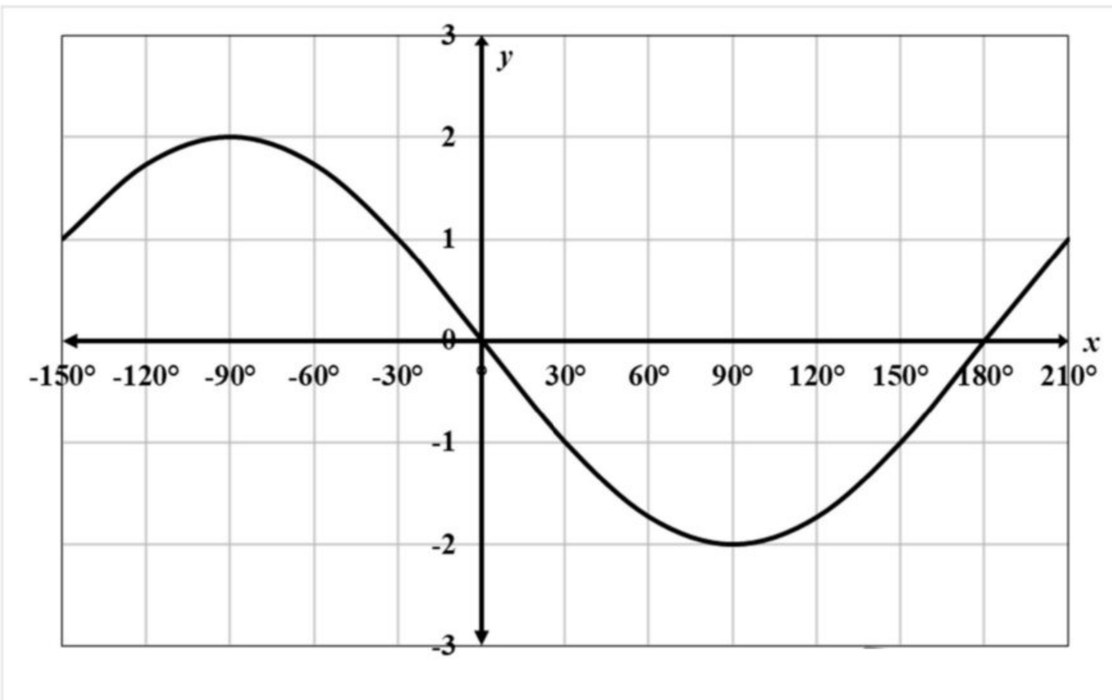
Given the equation: $\cos(x - 30°) + 2\sin x = 0$

6.1 Show that the equation can be written as $\tan x = -\frac{\sqrt{3}}{5}$. (4)

6.2 Determine the solutions of the equation $\cos(x - 30°) + 2\sin x = 0$ in the interval

$-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\sin x$ is drawn for $x \in [-150°; 210°]$.



6.3.1 Write down the amplitude of f. (1)

6.3.2 Draw the graph of $g(x) = \cos(x - 30°)$ for the interval $x \in [-150°; 210°]$ on the grid provided in the ANSWER BOOK. Clearly show ALL intercepts with the axes and endpoint(s) of the graph. (3)

6.3.3 Use the graphs to determine the values of x, in the interval $x \in [-150°\,;210°]$, for which:

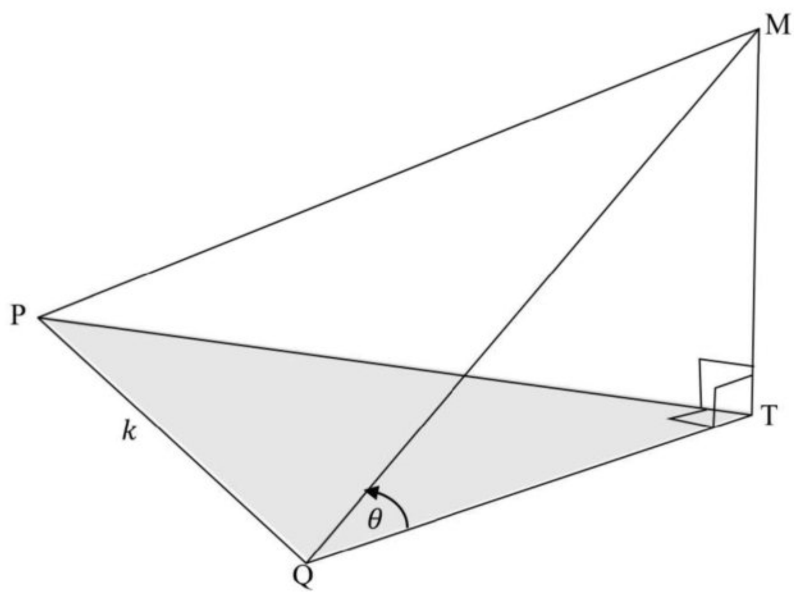
(a) $g(x) > f(x)$ (2)

(b) $f'\left(\frac{1}{2}x\right) = 0$ (1)

[14]

QUESTION 7

In the diagram below, P, Q and T are three points in the same horizontal plane and MT is a vertical mast. MP and MQ are two straight stay wires. The angle of elevation of M from Q is θ.
PQ = k metres, PM = 2PQ. The area of $\Delta MPQ = 2k^2 \sin\theta\cos\theta$.



7.1 Show that $M\hat{P}Q = 2\theta$. (3)

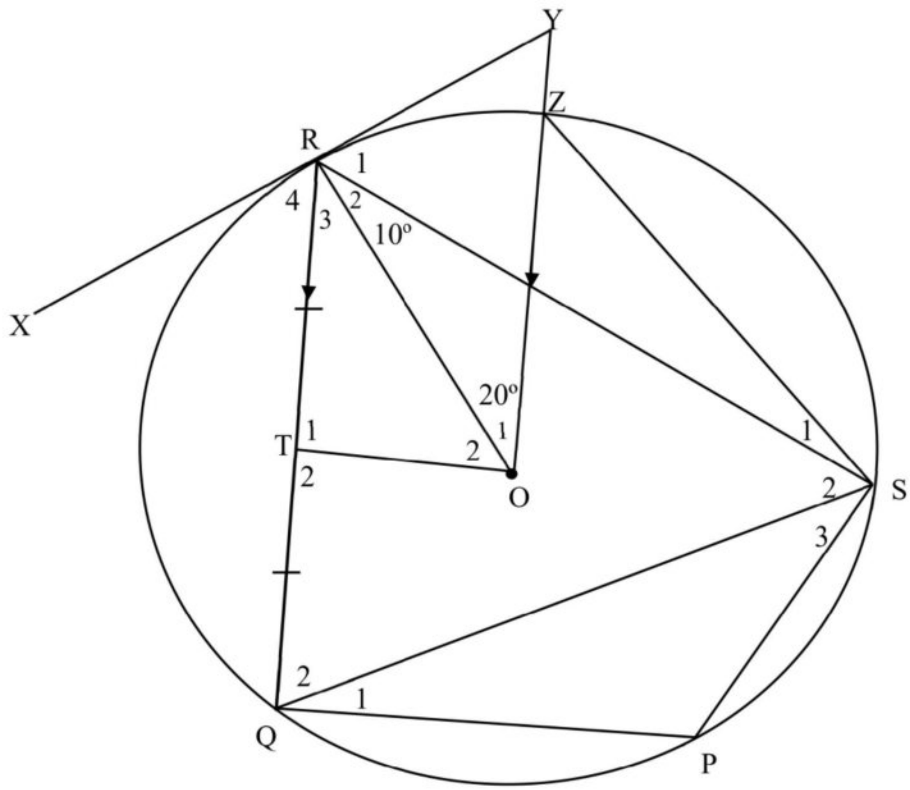
7.2 Hence, show that $MQ = k\sqrt{1+8\sin^2\theta}$. (4)

7.3 If k = 139,5 m and θ = 42°, determine the length of MT correct to the nearest metre. (3)

[10]

QUESTION 8

In the diagram below, points P, Q, R and S are points on a circle with centre O. OT bisects chord QR at T. XRY is a tangent to the circle at point R. OZ is produced to meet at Y where OY || QR.
$R\hat{O}Y = 20°$ and $S\hat{R}O = 10°$. Chord SZ is drawn.



8.1 Calculate, with reasons, the size of the following angles:

8.1.1 $\hat{S}_1$ (2)

8.1.2 $\hat{R}_3$ (1)

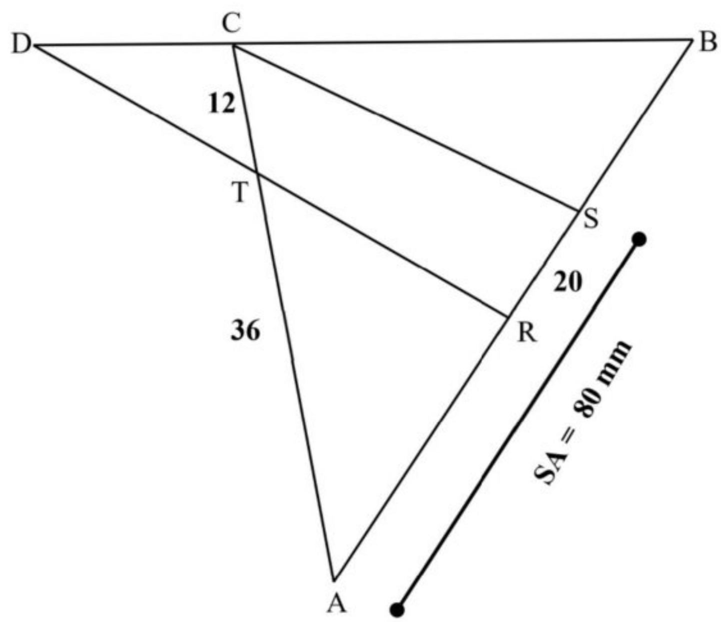
8.1.3 $\hat{P}$ (2)

8.1.4 $\hat{S}_2$ (4)

8.2 Prove that XRY is a tangent to the circle passing through R, T and O (3)
[12]

QUESTION 9

In the diagram below, ΔABC is constructed such that BC is produced to D. DR is drawn, with point T on AC and R on BA. CS is drawn. CT = 12 mm, TA = 36 mm, SR = 20 mm and SA = 80 mm.



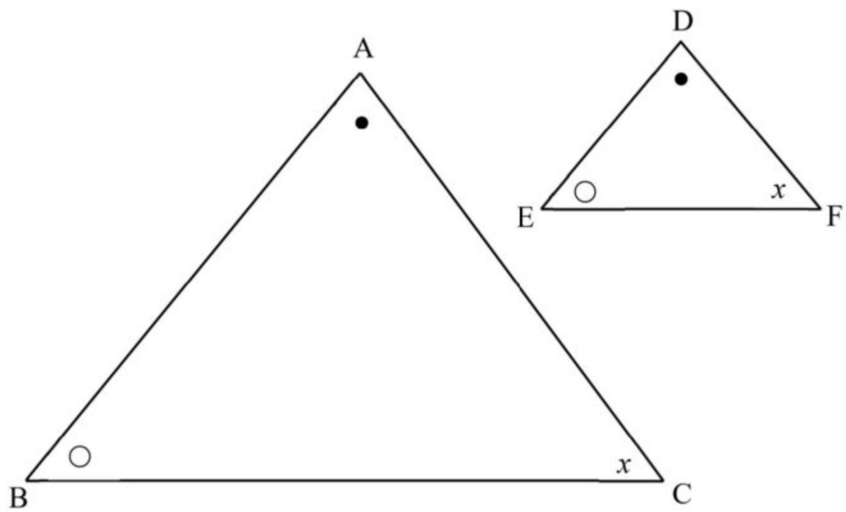
9.1 Prove that $CS \parallel TR$. (3)

9.2 It is further given that $AR = \frac{2}{3}RB$, $BC = 2x$ and $CD = \frac{1}{2}x + 1$.

Calculate the value of x. (6)
[9]

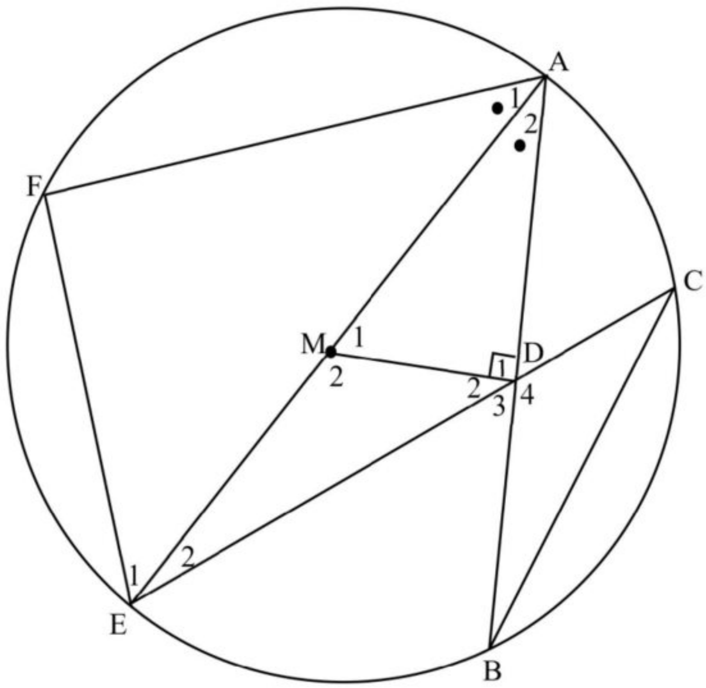
QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn, such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$. (6)

10.2 In the diagram below, diameter EMA of a circle with centre M bisects $F\hat{A}B$. MD is perpendicular to the chord AB. ED produced meets the circle at C. Chords CB and FE are drawn.



10.2.1 Prove that $\Delta AEF \parallel\!| \Delta AMD$. (4)

10.2.2 Determine the numerical value of $\frac{AF}{AD}$. (3)

10.2.3 Prove that $AD^2 = CD \times DE$. (6)

[19]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$