GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

# PREPARATORY EXAMINATION

# 2023

**11091**

**TECHNICAL MATHEMATICS**

**(PAPER 1)**

**TIME:** 3 hours

**MARKS:** 150

TECHNICAL MATHEMATICS: Paper 1

11091E

X05

**9 pages + a 2-page information sheet and 2 answer sheets**

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Answer QUESTIONS 4.2 and 5.4 on the ANSWER SHEETS provided. Please staple these ANSWER SHEETS to the FRONT of your ANSWER BOOK.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. Write neatly and legibly.

**QUESTION 1**

1.1 Given: $f(x) = x + 3$ and $g(x) = x^3 - 16x$

Solve for $x$ if:

1.1.1 $f(x) = 2$ (2)

1.1.2 $g(x) = 0$ (4)

1.1.3 $f(x) = x^2$ (correct to ONE decimal place) (4)

1.1.4 $-3 - x \geq x^2 - 5$ (4)

1.2 Solve for $x$ and $y$ simultaneously if $2^y - 16^x = 0$ and $y = x^2 + 4x - 4$. (6)

1.3 Express both the $x$ and $y$ values in QUESTION 1.2 (positive values) as binary numbers. (2)

1.4 Kinetic energy is the energy that an object has due to motion.

$$KE = \frac{1}{2}mv^2$$

$m$ = mass (measured in *kg*)
$v$ = velocity (measured in *m/s*)
$KE$ = Kinetic Energy (measured in *J*)

1.4.1 Write out the formula with $v$ as the subject of the formula. (3)

1.4.2 Determine the velocity of a 55 kg woman who is running, and her kinetic energy is 411,675 *J*. Round-off your answer to one decimal place. (2)

**[27]**

**QUESTION 2**

2.1 Without solving the equation, discuss the nature of the roots of:

$3x^2 - 5x + 1 = 0$ (2)

2.2 Determine the values of $k$ for which the equation $x^2 + (k + 2)x + 3k = 2$ has two distinct real roots. (5)

**[7]**

**QUESTION 3**

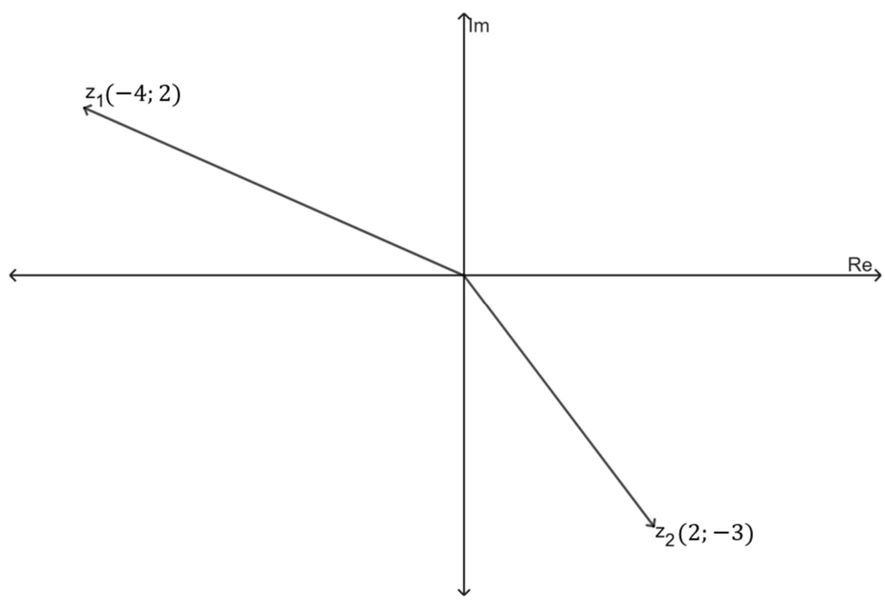
3.1 Simplify the following **without the use of a calculator**. (Show ALL calculations.)

3.1.1 $3\log 20 - \log 4 - \log 2$ (4)

3.1.2 $\dfrac{2i^8 - 3i^7 + 4i^6}{5i^9}$ (5)

3.1.3 $\dfrac{\sqrt{75} - \sqrt{27}}{\sqrt{48}}$ (3)

3.2 The Argand diagram shows points $z_1(-4; 2)$ and $z_2(2; -3)$.




3.2.1 Determine the sum of $z_1$ and $z_2$. (2)

3.2.2 Write down the conjugate of the sum determined in QUESTION 3.2.1. (1)

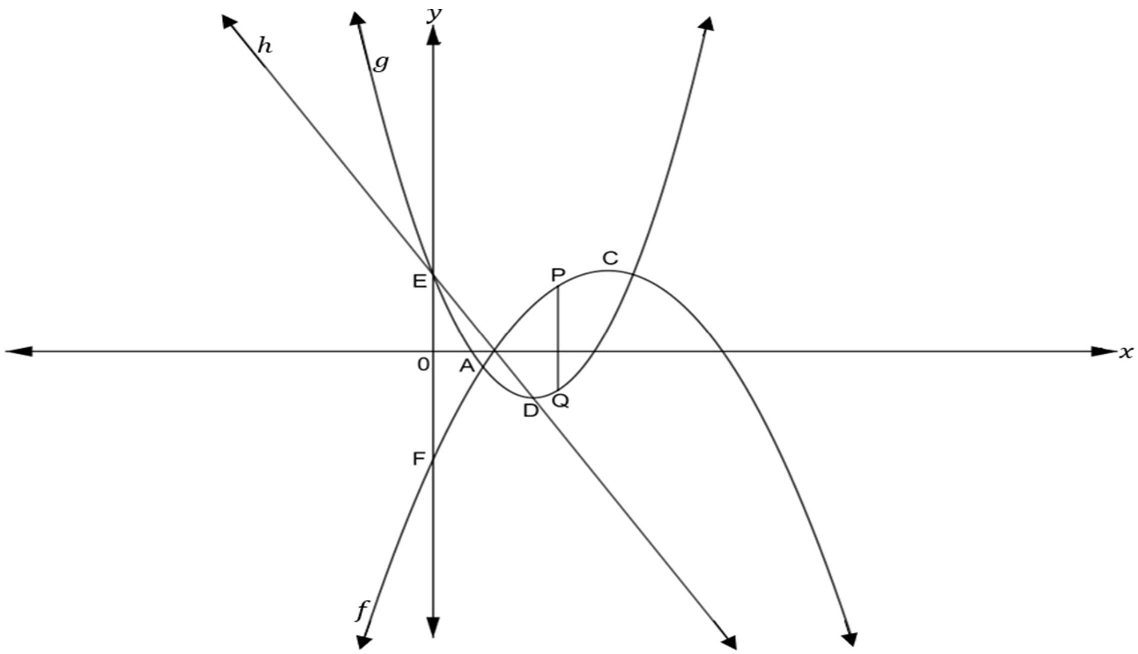
3.2.3 Draw both the sum and its conjugate on the same Argand diagram. (2)
**[17]**

**QUESTION 4**

The sketch below represents the functions defined by:

$f(x) = -x^2 + 7x - 7$, $g(x) = 2(x-2)^2 - 3$ and $h(x) = mx + k$

- The $y$-intercept of $g$ is (0; 5).
- C and D are the turning points of $f$ and $g$ respectively.
- The point E is the point of intersection of $g$ and $h$.




4.1 Determine:

4.1.1 The length of EF (2)

4.1.2 The coordinates of C and D, the turning points of $f$ and $g$, respectively (5)

4.1.3 The coordinates of Q if the coordinates of P are $\left(\frac{5}{2}; \frac{17}{4}\right)$ (3)

4.2 Use the attached ANSWER SHEET to draw the sketch graph of $h(x) = \frac{a}{x} + q$ indicating the intercepts and asymptote with the following conditions:

$a < 0$
$x \neq 0$
$y \neq 3$
$h(2) = 0$ (3)

**[13]**

## QUESTION 5

Given: $f(x) = 4^x - 4$ and $g(x) = -\sqrt{16 - x^2}$.

5.1 What type of graph is $f$? (1)

5.2 Calculate:

5.2.1 The $x$-intercept of $f$ (3)

5.2.2 The $y$-intercept of $f$ (2)

5.3 Write down the equation of the asymptote of $f$. (1)

5.4 Use the attached ANSWER SHEET to sketch the graphs of $f$ and $g$ on the same set of axes. Indicate ALL the intercepts with the axes and asymptotes. (7)

5.5 On your sketch in QUESTION 5.4, use the letter "**A**" to indicate where $f(x) = g(x)$. (1)

5.6 Write down the equation of $h$, if $f$ is shifted 5 units up. (1)

5.7 Write down the range of $g$. (2)

**[18]**

## QUESTION 6

6.1 The nominal interest rate charged on an investment is 7,2% p.a. compounded half-yearly. Calculate the annual effective interest rate charged per annum. (4)

6.2 Gomolemo buys a cellphone from Dikhudu which costs R3 400. Gomolemo is willing to pay off the cellphone over a period of one year.

Dikhudu calculates what he would receive from Gomolemo if he charged interest at 9,2% per annum, compounded daily (use 365 days).

6.2.1 Calculate how much Gomolemo must pay in total (to the nearest rand). (4)

6.2.2 Did Dikhudu offer Gomolemo a good deal? (Motivate your answer.) (2)

6.3 Poppie invested an amount of R20 000 for 6 years. She earns 12,5% interest per annum, compounded monthly for the first two years.

After that the interest rate changed to 9,8% per annum, compounded quarterly for the rest of the investment.

Determine the amount that she would have saved at the end of the period. (5)

**[15]**

**QUESTION 7**

7.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x) = 5 - 3x$. (4)

7.2 Determine:

7.2.1 $\lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2}$ (4)

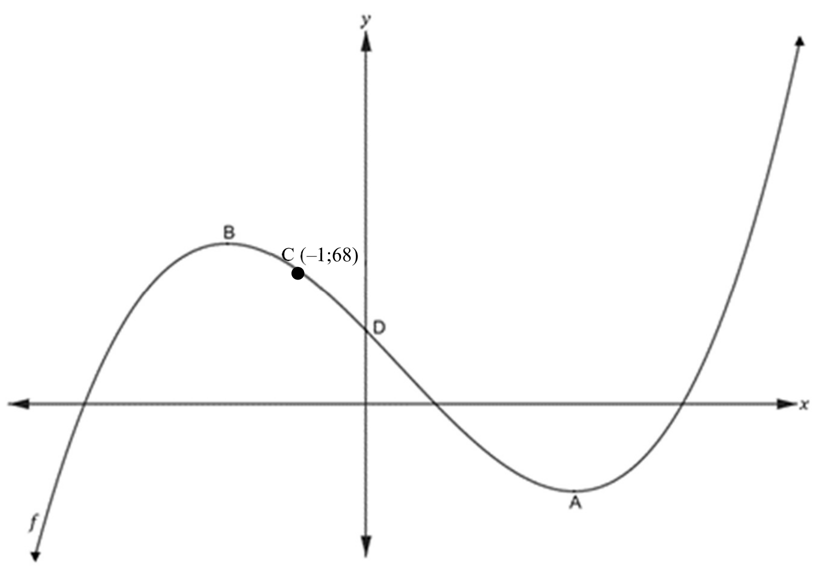
7.2.2 $\frac{d}{dx}[(x^2 + 1)(x^{-2} - 1)]$ (express your answer with positive exponents) (4)

7.3 Determine the equation of the tangent to the curve defined by $g(x) = 4x - x^2$ at the point where $x = 3$. (5)

**[17]**

**QUESTION 8**

The sketch below represents the curve of $f$ defined by $f(x) = 2x^3 - 3x^2 - 36x + 37$.
A and B are the turning points of $f$ and $C(-1; 68)$ is a point on $f$.




8.1 Determine ONE linear factor of $f(x) = 2x^3 - 3x^2 - 36x + 37$. (1)

8.2 Write down the coordinates of D. (1)

8.3 Determine the coordinates of B, the local maximum point of *f*. (5)

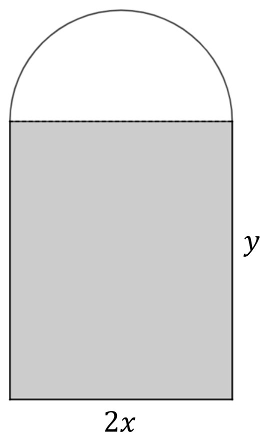
8.4 For which value(s) of $x$ is $f$ a decreasing function? (2)

8.5 Show that the equation of the tangent at C(–1; 68) is given by $y = -24x + 44$. (3)

**[12]**

**QUESTION 9**

A bathroom window is in the form of a rectangle with a semicircle at the top. The rectangle has the following dimensions: Breadth = $2x$ metres and length = $y$ metres.




The semicircle is made of clear glass while the rectangular part is made of coloured glass which transmits only half as much light per square metre as the clear glass. The total perimeter of the window is fixed at 6 metres.

**The following formulae may be used to assist you in answering this question:**

Area of rectangle = (length $\times$ breadth)
Area of circle = $\pi r^2$
Perimeter of rectangle = $2(\text{length} + \text{breadth})$
Circumference of circle = $2(\pi r)$

9.1 Show that the height of the rectangle is given by:

$$y = 3 - x - \frac{\pi r}{2}$$ (2)

9.2 Show that the amount of light being transmitted through the window is $L(x) = -x^2 + 3x$. (5)

9.3 Determine the values of $x$ and $y$ (in terms of $\pi$) when the light entering through the window, is maximised. (3)

9.4 Hence, determine the value of $x$ so that the greatest possible amount of light enters through the window. (2)

**[12]**

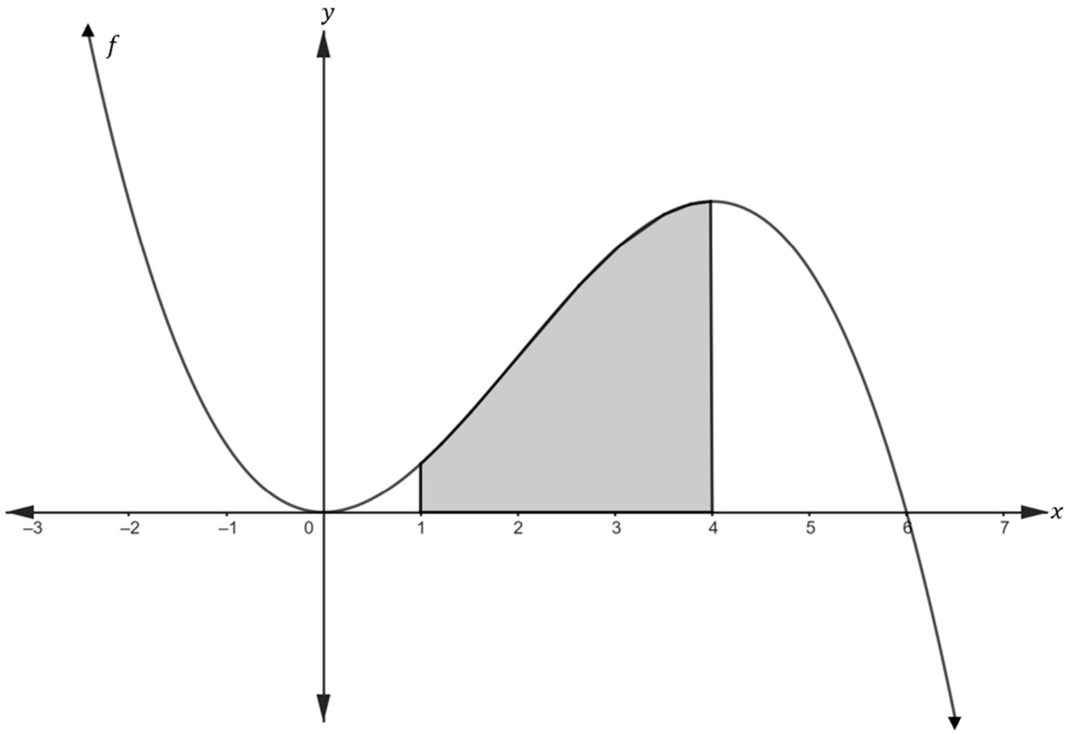
**QUESTION 10**

10.1 Determine:

10.1.1 $\int (4 + 2^{3x})dx$ (3)

10.1.2 $\int \left(\sqrt{x} + 6x^2 - \frac{8}{x}\right)dx$ (4)

10.2 The sketch below represents the shaded area bounded by the curve of the function defined by $f(x) = -x^3 + 6x^2$; and the $x$-axis between the points where $x = 1$ and $x = 4$.




Show that the area of the shaded region is not greater than $62\frac{1}{2}$ squared units. (5)

**[12]**

**TOTAL: 150**

END

## INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\frac{b}{2a}$ $\qquad y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = log_a b, \quad a > 0, a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 + i)^2$ $\qquad A = P(1 - i)^2$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C, n, k \in \mathbb{R}$ with $n \neq -1$ and $k \neq 0$

$\int \frac{k}{x} dx = kln x + C, x > 0$ and $k \epsilon \mathbb{R}\, ;\ k \neq 0$

$\int ka^{nx} dx = \frac{ka^{nx}}{n \ln a} + C, a > 0\ ; a \neq 1$ and $k, a \in \mathbb{R}\ ; k \neq 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\frac{x_1 + x_2}{2}\ ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad tan\theta = m$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In $\Delta$ABC: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ $\qquad a^2 = b^2 + c^2 - 2bc.cosA$

$Area\ of\ \Delta ABC = \frac{1}{2} ab.sinC$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n$ where $n$ = rotation frequency

Angular velocity $= \omega = 360^0 n$ where $n$ = rotation frequency

Circumferential velocity $= v = \pi Dn$ where $D$ = diameter and $n$ = rotation frequency

Circumferential velocity $= v = \omega r$ where $\omega$ = angular velocity and $r$ = radius

Arc length $= s = r\theta$ where $r$ = radius and $\theta$ = central angle in radians

Area of $a$ sector $= \frac{rs}{2}$ where $r$ = radius, $s$ = arc length

$Area\ of\ a\ sector = \frac{r^2\theta}{2}$ where $r$ = radius and $\theta$ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where $h$ = height of segment, $d$ = diameter of circle and $x$ = length of chord

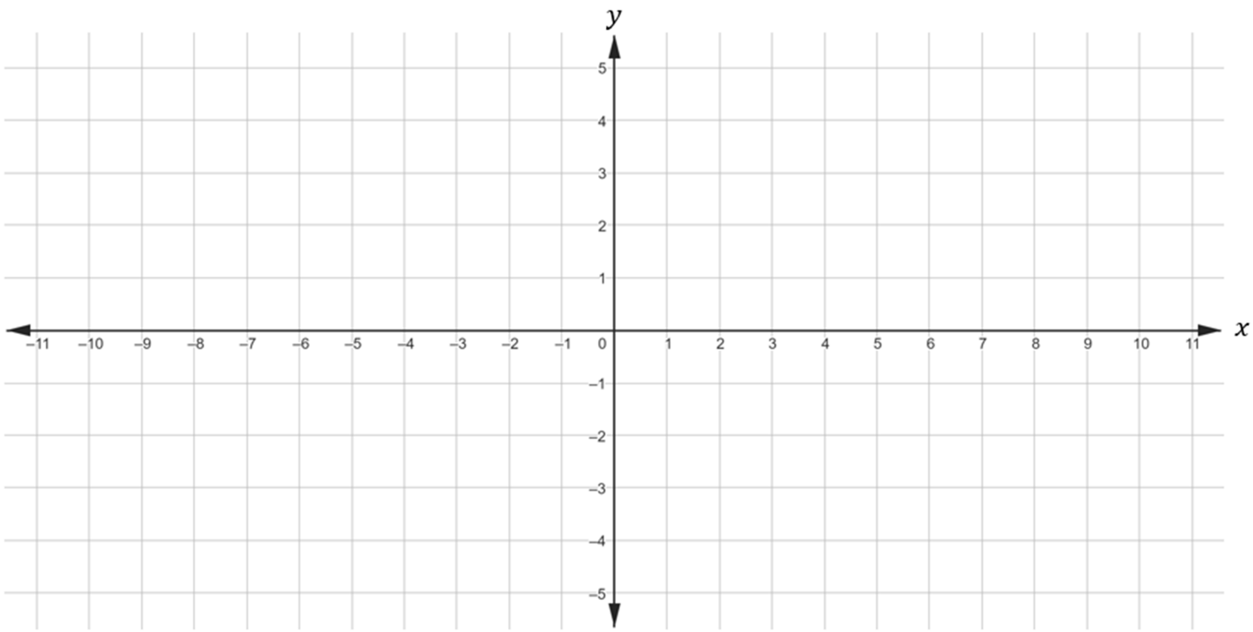
$A_T = a(m_1 + m_2 + m_3 + \cdots + m_n)$ where $a$ = number of equal parts, $m_1 = \frac{o_1+o_2}{2}$
$o_n = n^{th}$ ordinate and $n$ = number of ordinates

**OR**

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \cdots + o_{n-1}\right)$ where $a$ = number of equal parts, $o_n = n^{th}$ ordinate and $n$ = number of ordinates

**Candidate's name:** ____________________________________________

**QUESTION 4.2**



**Candidate's name:** ______________________________________________

**QUESTION 5.4**