

You have Downloaded, yet Another Great Resource to assist you with your Studies ©

Thank You for Supporting SA Exam Papers

Your Leading Past Year Exam Paper Resource Portal

Visit us @ www.saexampapers.co.za





# JUNE EXAMINATION GRADE 12

2024

# **MATHEMATICS**

(PAPER 2)

**MATHEMATICS P2** 

TIME: 3 hours

**MARKS: 150** 







MATHE	MATICS		2
(PAPER	. 2)	GR12 0624	_

#### INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

- 1. This question paper consists of 10 questions.
- 2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
- 3. Clearly show ALL calculations, diagrams, graphs etc. which you have used in determining your answers.
- 4. Answers only will NOT necessarily be awarded full marks.
- 5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
- 6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
- 7. Diagrams are NOT necessarily drawn to scale.
- 8. An information sheet with formulae is included at the end of the question paper.
- 9. Write neatly and legibly.





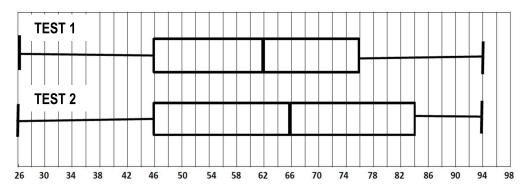
MATHEMATICS		3
(PAPER 2)	GR12 0624	_

1.1 The data below shows the marks (out of 100), obtained by 16 learners in a Mathematics test (Test 1).

Ī	67	77	26	92	48	38	56	58
	75	83	32	94	60	44	64	68

Calculate the:

- 1.1.1 Mean mark for the test (2)
- 1.1.2 Standard deviation of the data (1)
- 1.1.3 The number of learners whose marks are outside one standard deviation of the mean (2)
- 1.2 The 16 learners, referred to in QUESTION 1.1 above, wrote a second test (Test 2). The box and whisker diagram below shows the distribution of marks obtained by the 16 learners in the two tests.



- 1.2.1 Describe the skewness of the marks for Test 1. (1)
- 1.2.2 Which test was easier for the learners? Give a reason for your answer. (2)
- 1.2.3 How many learners scored more than 84% in the second test? (2) [10]

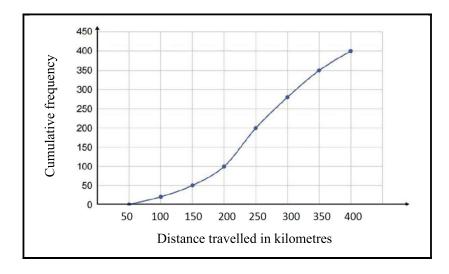




MATHEMATICS		4
(PAPER 2)	GR12 0624	·

2.1 In a survey, a group of people were asked about the total distance they had travelled from their homes in the last week of December 2023. The data collected is represented in the frequency table and ogive (cumulative frequency curve) below.

Distance travelled (x kilometres)	Frequency
$50 \le x < 100$	20
$100 \le x < 150$	30
$150 \le x < 200$	A
$200 \le x < 250$	В
$250 \le x < 300$	80
$300 \le x < 350$	70
$350 \le x < 400$	50



- 2.1.1 How many people participated in this survey? (1)
- 2.1.2 Determine the values of  $\mathbf{A}$  and  $\mathbf{B}$  in the table above. (2)
- 2.1.3 Use the ogive/table to estimate the number of people who travelled between 100 km and 300 km. (2)
- 2.1.4 If all people who travelled more than 350 km were removed from the survey, how would this affect the median of the data? (1)
- 2.2 If the estimated mean of the data below is 16,4, then what is the value of t?

Class Interval	Frequency
$0 < x \le 10$	13
$10 < x \le 20$	t
$20 < x \le 30$	12
$30 \le x < 40$	4

(4)

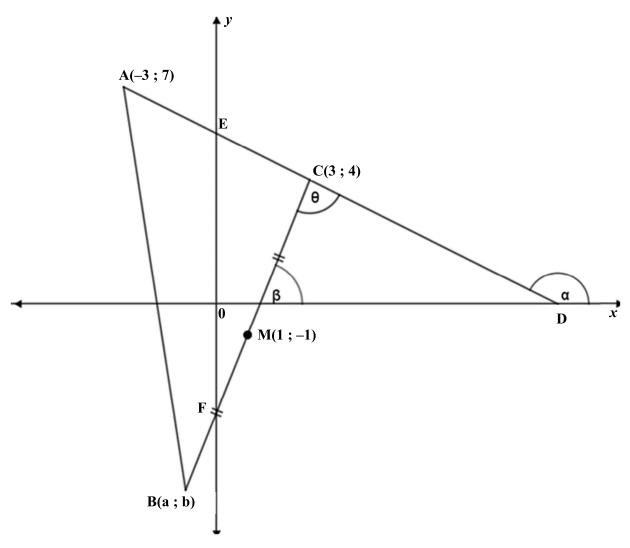
[10]





MATHEMATICS		5
(PAPER 2)	GR12 0624	

In the diagram below, the equation of line AD is 2y + x = 11. M(1; -1) is the midpoint of the straight line joining B(a; b) and C(3;4). The angles of inclination of AD and BC are  $\alpha$  and  $\beta$  respectively. E and F are the y-intercepts of AD and BC respectively.



- 3.1 Determine the coordinates of B(a; b). (3)
- 3.2 Determine the gradient of BC. (2)
- 3.3 Determine the size of  $\theta$ . (Correct to ONE decimal place) (4)
- 3.4 Determine the equation of line BC. (2)
- 3.5 Determine the area of  $\Delta$ CEF. (4) [15]

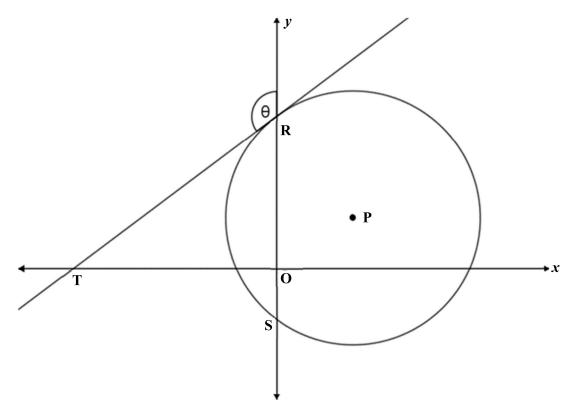




MATHEMATICS (PAPER 2)	GR12 0624	6
(I AI ER 2)	GK12 0024	

The diagram below shows a circle with centre P and equation  $x^2 + y^2 - 6x - 4y = 12$ .

The circle cuts the y-axis at points R and S. RT is a tangent to the circle at R and cuts the x-axis at T. Angle  $\theta$  is indicated.



- 4.1 Determine the coordinates of P as well as the radius of the circle. (4)
- 4.2 Show that the coordinates of R is R(0;6). (2)
- 4.3 Determine the equation of tangent RT in the form y = mx + c. (3)
- 4.4 Determine the size of  $\theta$ . (Correct to ONE decimal place) (3)
- 4.5 A vertical line is drawn as a tangent to circle P at Q(a; b) where a > 0. Write down the coordinates of Q. (2)
- 4.6 For which values of k will  $y = +\frac{3}{4}x + k$ , be a secant to the circle? (4)
- Another circle M with equation  $(x+3)^2 + (y+2)^2 = 36$  is drawn. Will circle M touch or cut circle P or will it neither touch nor cut circle P? Show all calculations to determine your answer. (4)





MATHEMATICS		7
(PAPER 2)	GR12 0624	,

5.1 Simplify the following expression, without using a calculator:

$$tan(-x) \cdot sin(90^{\circ} + x) + \frac{sin2x}{2cos(360^{\circ} + x)}$$
 (6)

5.2 Given that  $sin27^{\circ} = p$ , determine, without the use of a calculator, each of the following in terms of p.

$$5.2.1 \quad cos 27^{\circ}$$
 (2)

$$5.2.2 \quad \sin^2 63^{\circ}$$
 (2)

$$5.2.3 \quad cos13,5^{\circ}$$
 (3)

5.3 If cosx + sinx = k, express the following in terms k:

$$\cos\left(x-45^{\circ}\right) \tag{4}$$

5.4 Prove the identity:

$$\frac{\cos 2\theta + 1}{\sin 2\theta} + \tan \theta = \frac{1}{\sin \theta \cos \theta} \tag{5}$$

5.5 Determine the general solution of the equation:

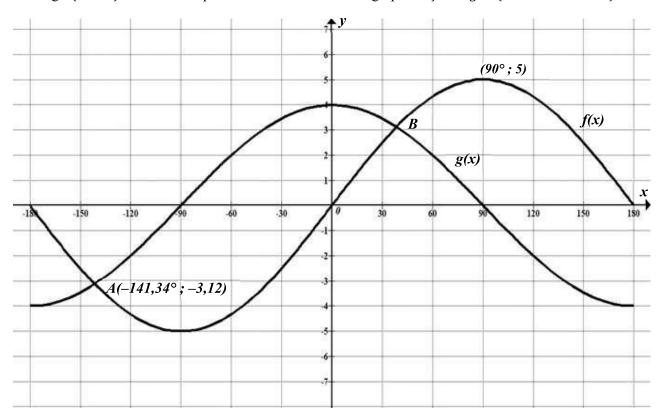
$$4\sin^2\theta = \cos(90^\circ - 2\theta) \tag{6}$$
[28]





MATHEMATICS	CD12.0624	8
(PAPER 2)	GR12 0624	

In the diagram below, f(x) = asinbx and g(x) = 4cosx are drawn for the interval  $x \in [-180^\circ; 180^\circ]$ . The graph of f(x) passes through  $(90^\circ; 5)$  and the graph of g(x) passes through  $(90^\circ; 0)$ . A and B are points of intersection of the graphs of f and g. A $(-141,34^\circ; -3,12)$ .



- 6.1 Determine the values of a and b. (2)
- 6.2 The graph of g(x) is shifted 30° to the right and 2 units vertically down to form the graph of h(x). Determine the equation of h(x). (2)
- 6.3 Calculate the minimum value of  $\frac{8}{g(x)}$  in the interval  $x \in [-60^\circ; 60^\circ]$ . (1)
- 6.4 If  $A(-141,34^{\circ}; -3,12)$ , determine the coordinates of B. (2)
- 6.5 For what values of k will f(x) = k have no real solutions? (2)
- 6.6 Use the graph(s) to determine the values of x in the interval  $x \in [-180^{\circ}; 180^{\circ}]$  for which:

6.6.1 
$$f(x) < g(x)$$
 (2)

6.6.2 
$$f'(x) \cdot g(x) \ge 0$$
 (2) [13]

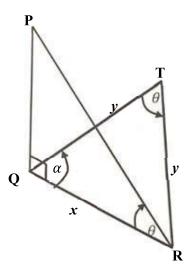




MATHEMATICS (PAPER 2)	GR12 0624	9
--------------------------	-----------	---

In the diagram below, TQR represents three points in a horizontal plane on a sportsfield. PQ represents a vertical flagpole.

The angle of elevation of the top of the pole from R is equal to  $\theta$ .  $\widehat{T} = \theta$ . TQ = TR = y. QR = x units.  $T\widehat{Q}R = \alpha$ .



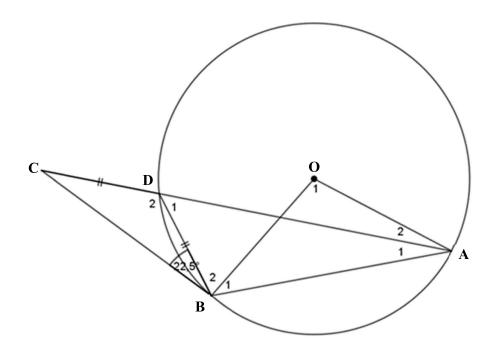
- 7.1 Express  $\theta$  in terms of  $\alpha$  and show that  $\sin \theta = \sin 2\alpha$  (2)
- 7.2 Hence, prove that in  $\triangle PQR$ :  $PR = \frac{2y\cos\alpha}{\cos\theta}$  (4)
- 7.3 If  $\alpha = 49^\circ$ ; x = 20 m and y = 15 m, calculate the area of  $\Delta TQR$ . [9]





MATHEMATICS (PAPER 2)	GR12 0624	10
,		

In the diagram below, O is the centre of the circle. A, B and D are points on the circle. BC is a tangent to the circle at B and  $D\hat{B}C = 22,5^{\circ}$ . Chord AD is produced to C such that CD = BD.



8.1 Determine, giving reasons, the size of:

8.1.1 
$$\hat{A}_1$$
 (2)

8.1.2 
$$\hat{D}_1$$
 (2)

8.1.3 
$$\hat{B}_2$$
 (2)

$$8.1.4 \quad \hat{O}_1$$
 (2)

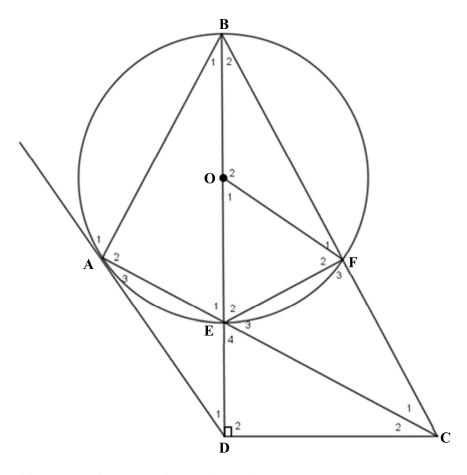
- 8.2 Prove, giving reasons, that  $OA \parallel CB$ . (2)
- 8.3 If it is further given that the radius of the circle is 12 units, calculate the length of BC. (4) [14]





MATHEMATICS		11
(PAPER 2)	GR12 0624	11

In the diagram below, the diameter BE of circle O is produced to D. DA is a tangent to the circle and  $CD \perp BD$ . AC and BC cut the circle at E and F respectively. OF and EF are drawn.



- Prove, with reasons, that ABCD is a cyclic quad. 9.1 (3)
- Prove, with reasons, that BD bisects ABC. 9.2 (3)
- 9.3 Prove, with reasons, that EC is a tangent to circle OEF. (4)

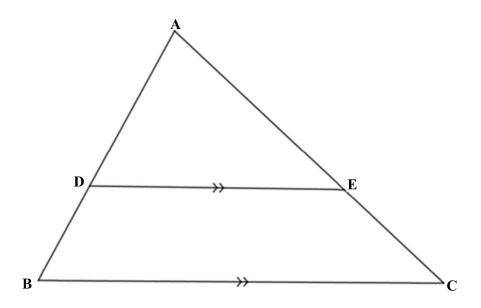
[10]





MATHEMATICS		12
(PAPER 2)	GR12 0624	12

10.1 In the diagram below,  $\triangle ABC$  is drawn with DE  $\parallel$  BC.



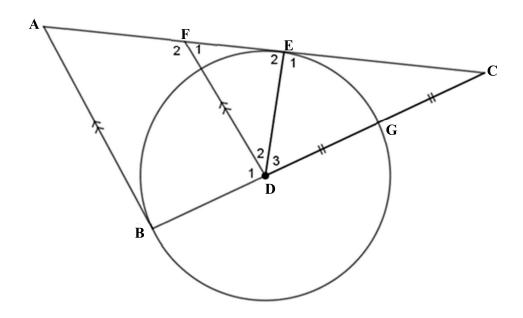
Prove the theorem which states that 
$$\frac{AD}{DB} = \frac{AE}{EC}$$
. (5)





MATHEMATICS		13
(PAPER 2)	GR12 0624	

10.2 In the diagram below, D is the centre of a circle. AB and AE are tangents to the circle at B and E respectively. The diameter BG is produced and meets tangent AE at C. DG = CG. F is a point on AC such that  $DF \parallel AB$ .



Find, with reasons, the ratio of 
$$\frac{AC}{FC}$$
. (3)

10.2.2 Prove, giving reasons, that 
$$\triangle ABC /// \triangle DEC$$
. (3)

10.2.3 Prove that 
$$DE^2 = \frac{AE.EC}{3}$$
 (5)

10.2.4 Find the ratio of: 
$$\frac{\text{Area}\Delta FDC}{\text{Area}\Delta ABC}$$
 (3)

**TOTAL: 150** 





**MATHEMATICS** (PAPER 2)

GR12 0624

14

#### INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1+ni)$$
  $A = P(1-ni)$   $A = P(1-i)^n$ 

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$T_n = a + (n-1)d$$
  $S_n = \frac{n}{2}[2a + (n-1)d]$ 

$$T_n = ar^{n-1}$$

$$T_{n} = ar^{n-1}$$

$$S_{n} = \frac{a(r^{n} - 1)}{r - 1} ; r \neq 1$$

$$S_{\infty} = \frac{a}{1 - r}; -1 < r < 1$$

$$P = \frac{x[(1 + i)^{n} - 1]}{i}$$

$$S_{\infty} = \frac{a}{1 - r}$$
;  $-1 < r < 1$ 

$$F = \frac{x[(1+i)^n - 1]}{x}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$
  $y - y_1 = m(x - x_1)$   $m = \frac{y_2 - y_1}{x_2 - x_1}$   $m = \tan \theta$ 

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In 
$$\triangle ABC$$
: 
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$area \ \Delta ABC = \frac{1}{2}ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta \qquad \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha . \cos \beta - \cos \alpha . \sin \beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\hat{v} = a + bx$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{r^2}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



