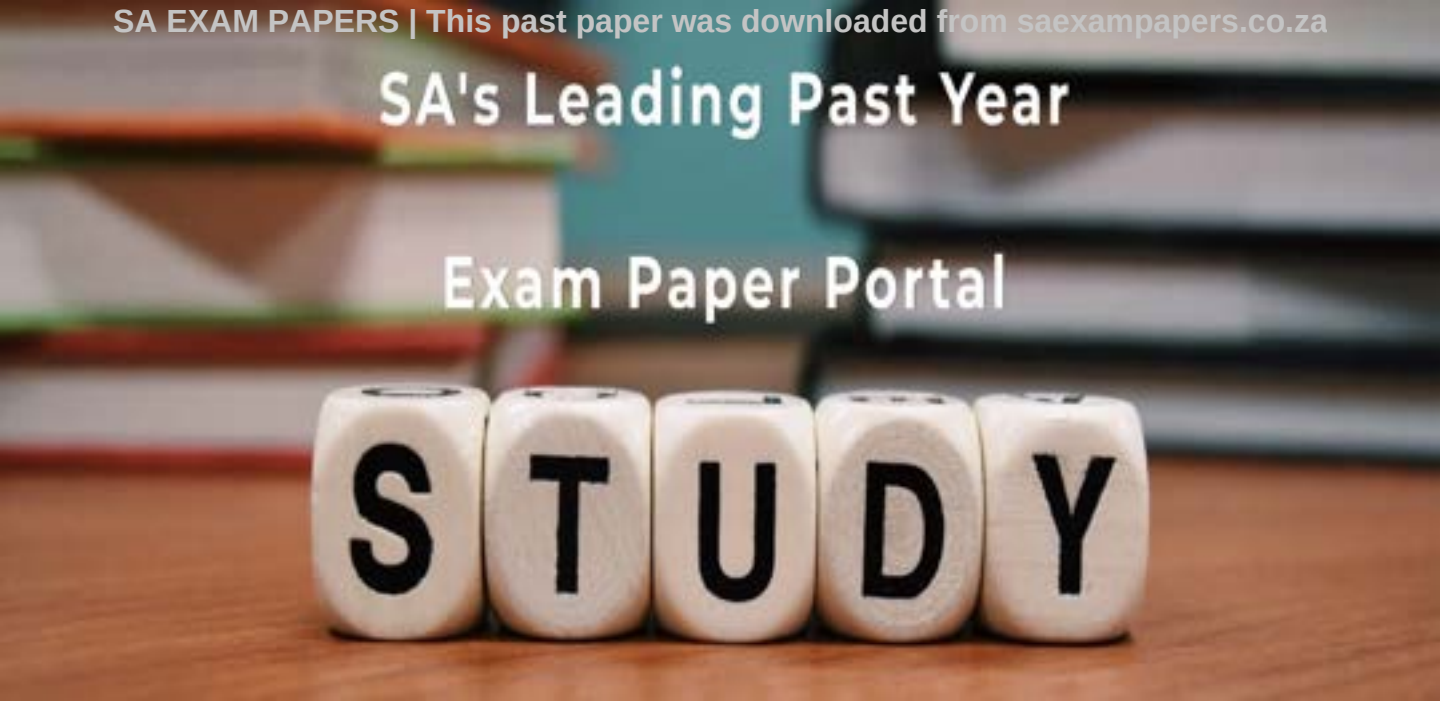


Province of the
EASTERN CAPE
EDUCATION

Iphondo leMpuma Kapa: Isebe leMfundo
Provinsie van die Oos Kaap: Department van Onderwys
Porafensie Ya Kapa Botjahabela: Lefapha la Thuto

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2024

TECHNICAL MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

This question paper consists of 13 pages, including a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 4.1.4 and QUESTION 7.4 on the ANSWER SHEETS provided. Write your name, surname and class in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $-x(x+9)=0$ (2)

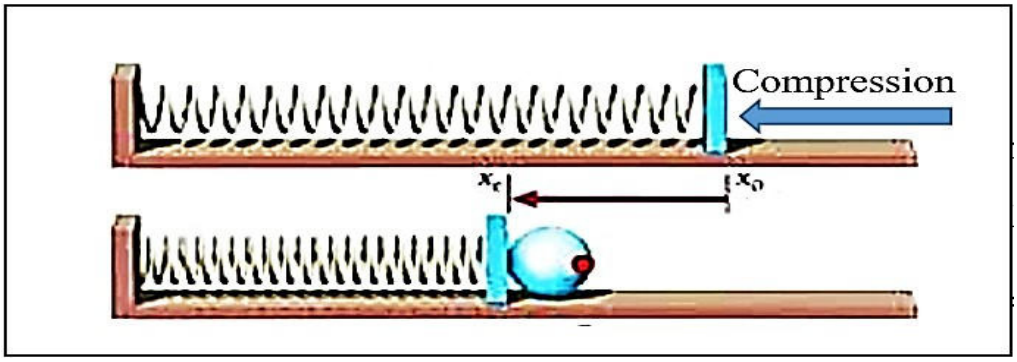
1.1.2 $x-\frac{2}{x}=0$ (Correct to ONE decimal place) (3)

1.1.3 $2x^2+7\leq 9x$ (4)

1.2 Solve for x and y if:

$2x+6y=-12$ and $-2xy+4y^2-10=0$ (6)

1.3 The diagram below shows a massless spring is compressed a distance x on a frictionless surface, resulting to work (W) done in Joules (J). The formula for the work done is denoted by: $W=\frac{1}{2}kx^2$, where k is the compression coefficient in N/m.



1.3.1 Make x the subject of the formula. (2)

1.3.2 Calculate the distance x after the compression, if the work done is 250 Joules (J) and the compression coefficient is 200 N/m. (2)

1.3.3 Write the answer in QUESTION 1.3.2 in scientific notation. (1)

1.4 Given the two decimal numbers X = 45 and Y = 15

1.4.1 Simplify: X – Y. (1)

1.4.2 Hence, write the answer in QUESTION 1.4.1 as a binary number. (2)

[23]

QUESTION 2

2.1 Given: $z = \dfrac{\sqrt{-x+3}}{x+9}$

Determine the value of x for which z:

2.1.1 Is undefined (2)

2.1.2 Has non-real roots (2)

2.2 $f(x) = -3x^2 + 2qx - 1$

Determine the value of q for which $f(x)$ will have unequal roots. (3)

[7]

QUESTION 3

3.1 Simplify the following, **without using a calculator:**

3.1.1 $\log_2 64$ (Show your working.) (2)

3.1.2 $$\frac{2^{3x-3}.3^{-x-1}}{\left(\frac{1}{4}\right)^{-x}.(2\times 3)^{x-3}.9^{-x+1}}$$ (5)

3.1.3 $$\frac{\sqrt{63}-2\sqrt{112}}{\sqrt{28}}$$ (3)

3.2 Solve for x: $\log_2(x-1) + \log_2(x+1) = 3$ (4)

3.3 Given: $z = 2\text{cis}\,\frac{\pi}{3}$

3.3.1 Write the argument in degrees. (1)

3.3.2 Write z in the rectangular form. (***Hint*: $z = a + bi$**) (3)

3.4 Solve for p and q:

$p + qi\,(2 - 3i) = 5 - 14i$ (5)

[23]

QUESTION 4

4.1 Given the functions f and g defined by $f(x) = (x + 1)^2 - 16$ and $g(x) = \sqrt{9 - x^2}$.

4.1.1 Write down the coordinates of the turning point. (2)

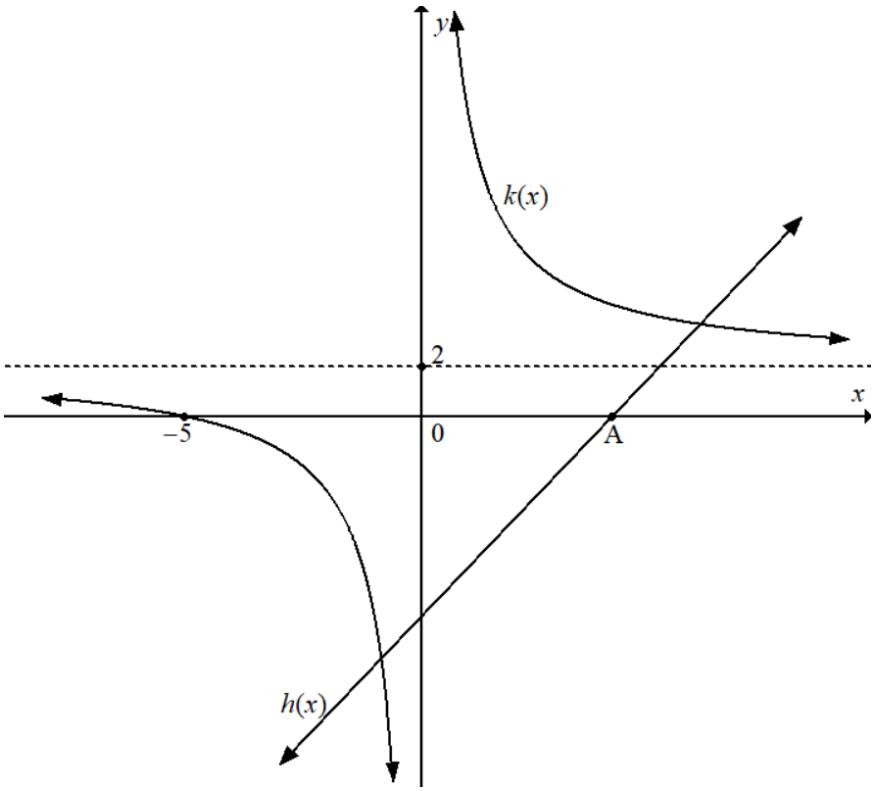
4.1.2 Determine the x-intercept of $f(x)$. (3)

4.1.3 Write the range of $g(x)$. (2)

4.1.4 Sketch the graph of $f(x)$ and $g(x)$ on the ANSWER SHEET provided. Clearly show all the intercepts and turning point with the axes. (5)

4.1.5 Hence or otherwise, determine the values of x for which $f'(x) > 0$ (2)

4.2 The graph of $k(x)$ and $h(x)$ defined by $k(x) = \frac{m}{x} + q$ and $h(x) = 2x - 8$ are sketched below. $x = -5$ and A are the x-intercepts of k and h, respectively. The asymptote of k cuts the axis at $y = 2$



4.2.1 Write the coordinates of point A. (2)

4.2.2 Write down the value of q. (1)

4.2.3 Hence, determine the value of m. (3)

4.2.4 Determine values of x for which $k(x) = h(x)$ (6)

4.3 Given a graph defined by $y = a^x + q$, with the following features.

- $0 < a < 1$
- Range: $\{y : y > 1;\ y \in \mathrm{R}\}$
- Domain: $x \in \mathrm{R}$
- Passes through $(-1;4)$

Write the complete equation of y. (3)

[29]

QUESTION 5

5.1 The effective interest rate is 9,13% per annum. Calculate the nominal interest rate compounded quarterly. (3)

5.2 Quinton buys a treadmill costing R24 000. He pays 20% deposit and then takes out a 24-month hire purchase loan on the balance. The interest rate charged on the loan is 12% per annum simple interest.

Calculate:

5.2.1 The monthly instalments (5)

5.2.2 The actual amount paid for the treadmill (1)

5.3 Calculate the monthly compound interest that makes R3 000 double in 6 years. (3)

5.4 A Master's degree student opened a savings account 10 years ago with an initial deposit of x rands. Four years after the initial deposit she made a further deposit of R15 000. The amount deposited earned 12% per annum compounded semi-annually for the first 5 years. The interest rate changed to 13,2% per annum compounded monthly thereafter.

Calculate the amount of money that she deposited initially if her savings account is worth R155 000 after 10 years. (5)

[17]

QUESTION 6

6.1 Determine the derivative of $f(x) = -\frac{1}{3}x - 4$. (5)

6.2 Determine:

6.2.1 $D_x\left[x^5 + \frac{x^{-4} - 2x}{2x^2}\right]$ (5)

6.2.2 $\frac{dy}{dx}$ as $y = ax^4 + \sqrt[3]{8x}$ (3)

6.3 The gradient of the tangent to a curve defined by: $f(x) = kx^2 - 4x + 5$ is equal to 16 at $x = -2$.

Determine the value of k. (3)

[16]

QUESTION 7

Consider: $f(x) = (x - 1)(x^2 - 7x + 6)$

7.1 Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$. (1)

7.2 Determine the coordinates of the x-intercepts of f. (3)

7.3 Determine the coordinates of turning points of f. (5)

7.4 Sketch the graph of $f(x)$ on the ANSWER SHEET provided. Clearly show all the intercepts with the axes and the turning points. (4)

7.5 Hence or otherwise, write down the values of x for which $f(x) \geq 0$. (2)

[15]

QUESTION 8

The temperature of a liquid during a certain experiment is given by:

$$T(t) = -2t^2 + 9,2t + 2$$

8.1 Determine the temperature of the liquid at the beginning of the experiment. (1)

8.2 Determine the rate of change of the temperature of the liquid during the first 3 seconds. (3)

8.3 After how many seconds did the liquid reach a maximum temperature? (2)

8.4 What was the maximum temperature reached. (1)

[7]

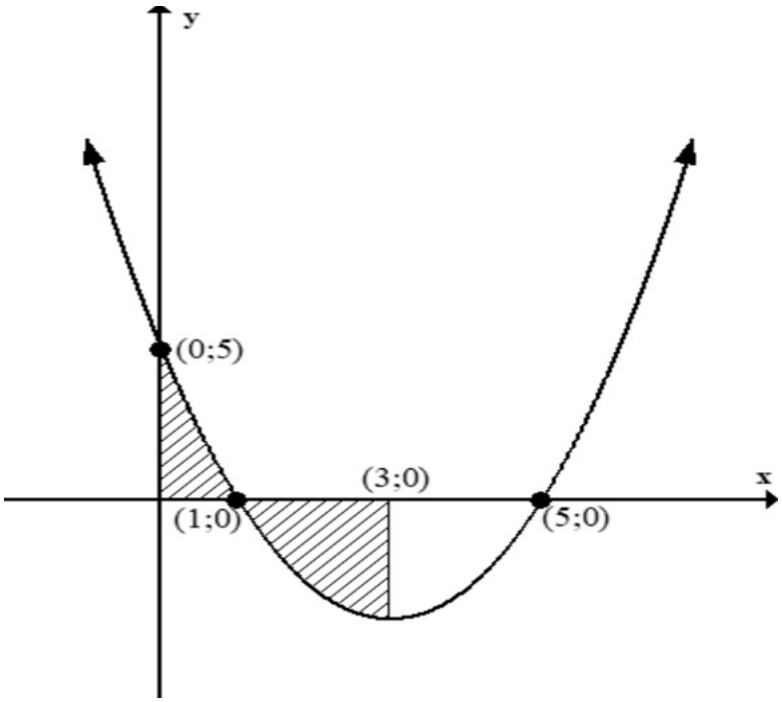
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int -4^{2t}\ dt$ (2)

9.1.2 $\int x\left(2x^{-1}+\frac{3}{x^2}\right)dx$ (4)

9.2 The diagram below shows the shaded area bounded by the function f defined by $f(x)=x^2-6x+5$ and the x-axis between the points where $x = 0$ and $x = 1$ together with $x = 1$ and $x = 3$.



Determine the area of the shaded region of the graph of h bounded by the graph and the x-axis, between $x = 0$ up to $x = 3$. (7)

[13]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1 \qquad \int k x^n dx = k.\frac{x^{n+1}}{n+1} + C, \; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \; x > 0 \qquad \int \frac{k}{x} dx = k.\ln x + C, \; x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \; a > 0 \qquad \int k a^{nx} dx = k.\frac{a^{nx}}{n \ln a} + C, \; a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$$

$\pi rad = 180°$ $v = 2\pi rn$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency
Angular velocity $= \omega = 360°n$ where n = rotation frequency

Circumferential velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency
Circumferential velocity = $v = 2\pi rn$ where r = radius and n = rotation frequency

Arc length = $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2}$ where r = radius, s = arc length and θ = central angle in radians

Area of a sector $= \frac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

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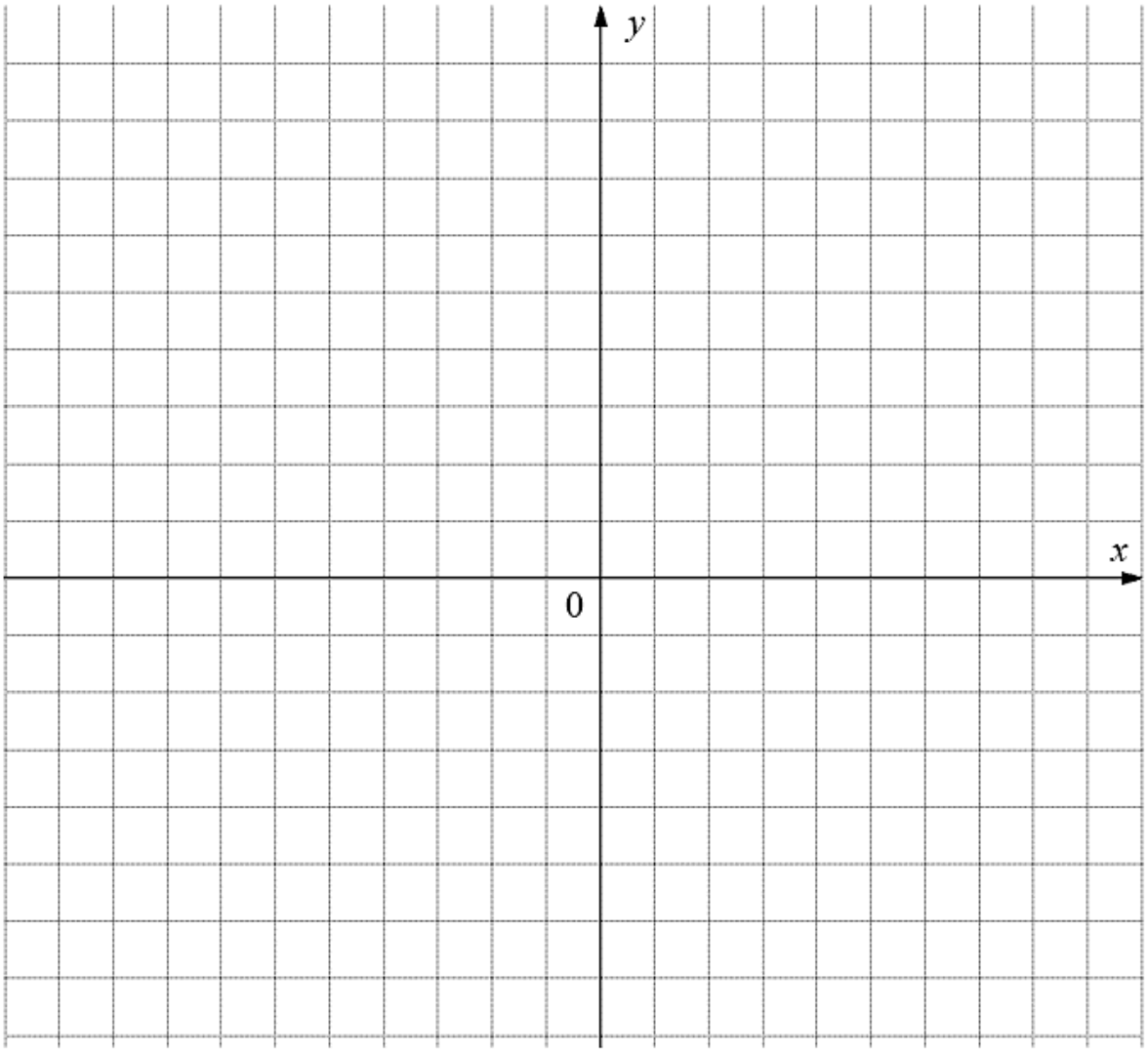
QUESTION 4.1.4



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QUESTION 7.4