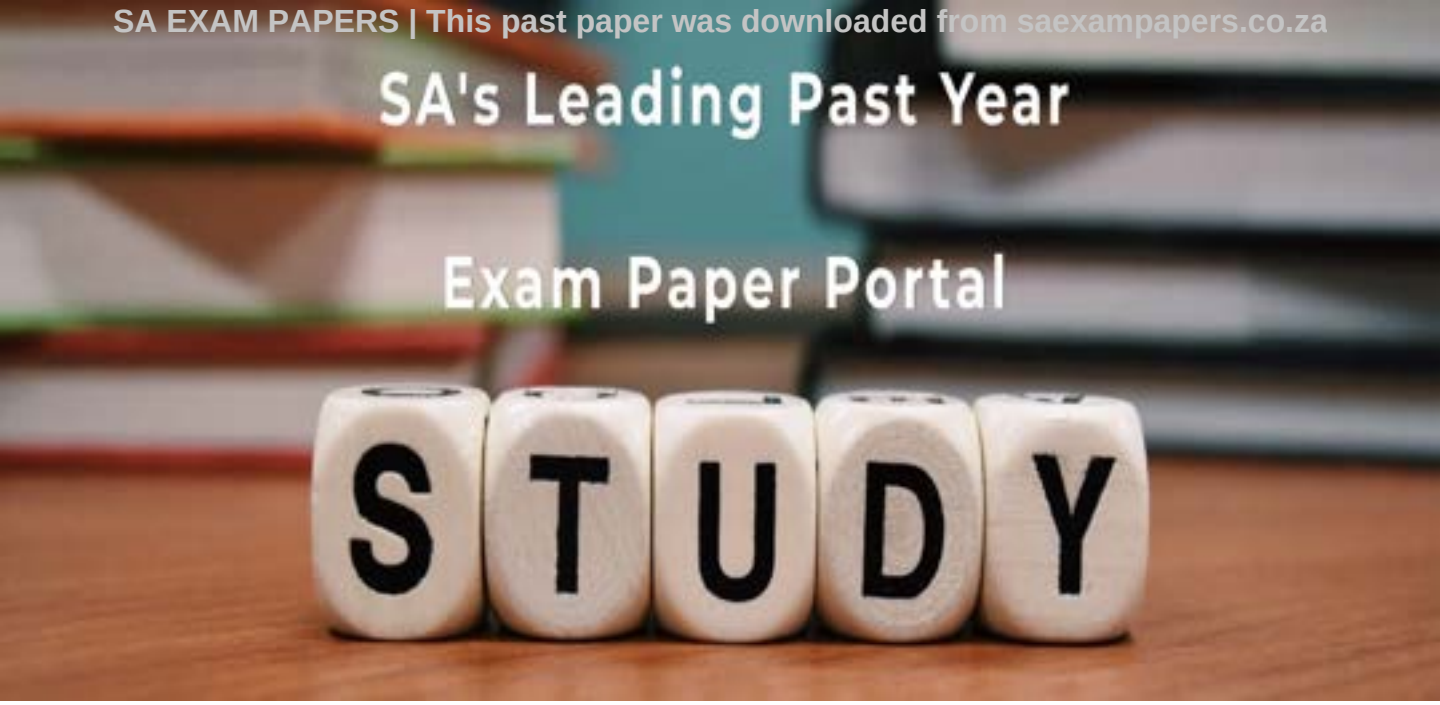


GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2024

11091

TECHNICAL MATHEMATICS

(PAPER 1)

TECHNICAL MATHEMATICS: Paper 1

11091E

TIME: 3 hours

MARKS: 150

8 pages + a 2-page information sheet and 2 answer sheets

X05

P.T.O.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
5. Answers ONLY will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Answer QUESTION 4.1.3 and QUESTION 7.5 on the ANSWER SHEETS provided. Write your name in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x + 6)(x - 5) = 0$ (2)

1.1.2 $-5x - 8 + 4x^2 = 0$ (Correct to TWO decimal places) (3)

1.1.3 $x^2 \leq 3x$ (3)

1.2 Solve for x and y simultaneously if:

$x + 2y = 3$ and $3x^2 + 4xy + 2y^2 = 9$ (6)

1.3 Acceleration (a) is the change in velocity (v) over the change in time (t), and is represented by the equation:

$a = \frac{v_2 - v_1}{t}$, where
$a =$ acceleration in m/s^2;
$v_1 =$ initial velocity measured in m/s;
$v_2 =$ final velocity measured in m/s; and
$t =$ time measured in s.

1.3.1 Make v_1 the subject of the formula. (2)

1.3.2 A sprinter increases his speed from x m/s to 15 m/s in 2 seconds. The acceleration of the sprinter is 2,5 m/s^2.

Determine the sprinter's speed at the beginning of the sprint in m/s. (2)

1.3.3 The final speed of the sprinter is 15 m/s.

Express this number as a binary number (ignore the units). (2)
[20]

QUESTION 2

2.1 Given: $\text{M} = \frac{\sqrt{25 - 2x}}{x}$

Determine the value(s) of x for which M will be:

2.1.1 Undefined (1)

2.1.2 Real (2)

2.2 Without solving the following equation, determine the nature of the roots of $6x^2 - 6x + 1 = 0$. (3)
[6]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $81^{-\frac{3}{4}}$ (3)

3.1.2 $\dfrac{2^{x+1}-3.2^{x-1}}{\left(\frac{1}{2}\right)^{-x}}$ (5)

3.2 Solve for x:

$\log_4(x+2) - \log_4 3 = 2$ (3)

3.3 Given: $z = \frac{-3+6i}{3}$

3.3.1 Write down the conjugate of z. (2)

3.3.2 Express $z = \frac{-3+6i}{3}$ in polar form. (6)

3.4 Solve for x and y if: $(x+yi) = (2-i)(3i+2)$ (4)

[23]

QUESTION 4

4.1 Given: $f(x) = x + 1$ and $g(x) = \frac{-4}{x} - 3$.

4.1.1 Write down the equation of the asymptotes of $g(x)$. (2)

4.1.2 Explain why $g(x)$ does not have a y-intercept. (2)

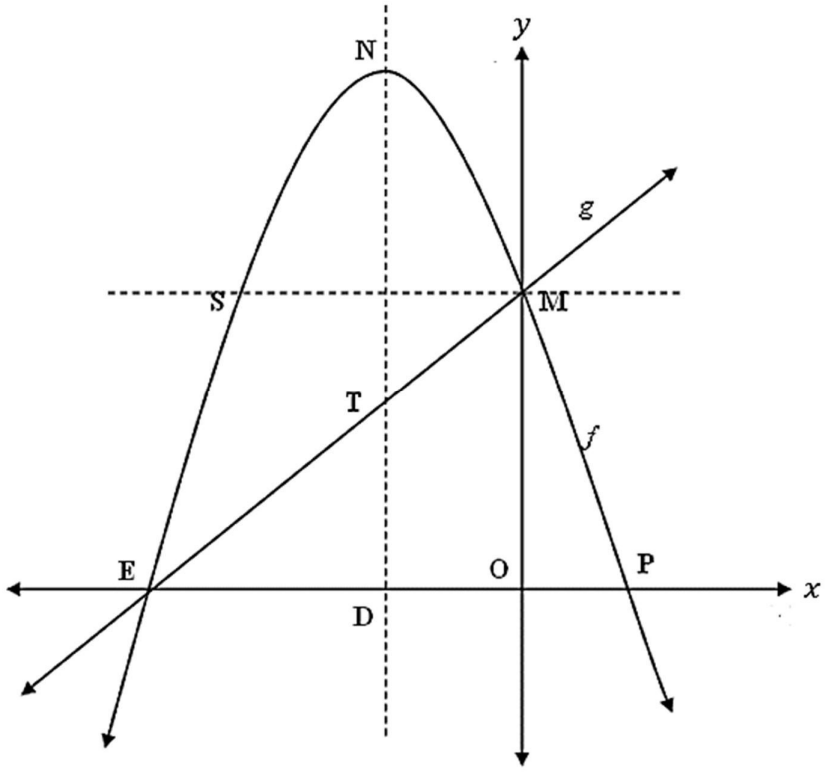
4.1.3 Sketch the graphs of $f(x)$ and $g(x)$ on the system of axes on the ANSWER SHEET provided. Clearly show all the intercepts with the axes and the asymptotes. (6)

4.1.4 Solve for x if $f(x) = g(x)$. (4)

4.1.5 Is the graph of $f(x)$ a tangent to the graph of $g(x)$ when $f(x) = g(x)$? Give a reason for your answer. (2)

4.2 The graph below represents the functions defined by $f(x) = -x^2 - 4x + 5$ and $g(x) = mx + c$.

- N is the turning point of $f(x)$.
- E and M are the points of intersections of f and g with the x and y-axes respectively.
- E and P are points on the x-axis.



Determine:

4.2.1 The coordinates of E and P (5)

4.2.2 The coordinates of M (2)

4.2.3 The coordinates of N, the turning point of f (4)

4.2.4 The equation of $g(x) = mx + c$ (2)

4.2.5 The length of NT (3)

4.2.6 The value(s) of x for which $f(x) > g(x)$ (2)

[34]

QUESTION 5

5.1 Convert a nominal interest rate of 12,3% p.a. compounded quarterly, to an effective annual interest rate. (3)

5.2 High School Proud Learners bought machinery for R500 000 a few years ago. It depreciates at a rate of 12% per annum on the diminishing-balance method. After how many years will the machinery's value be R150 000? (4)

5.3 Thabo, a mechanical engineer, invests R182 000 in a savings account at 7,2% p.a. compounded semi-annually. He wants to have sufficient money to start his own business in 5 years' time. He projects that he will need at least R300 000 as start-up capital. Two years after the initial deposit, he deposits a further R50 000 into the same savings account. At the end of the third year, he withdraws an amount of R 30 000 to pay for an urgent domestic expense. For the remainder of the investment period the interest rate changes to 7% p.a., compounded quarterly. Determine whether Thabo will have sufficient funds to start his own business in 5 years' time. (7)

[14]

QUESTION 6

6.1 Determine $f'(x)$, using FIRST PRINCIPLES if $f(x) = -x - 6$. (4)

6.2 Determine the following:

6.2.1 $f'(x)$ if $f(x) = \sqrt[3]{x^3} + 2x^{-2} - \frac{1}{2x}$ (4)

6.2.2 $\frac{d}{dx}\left[\frac{2x^2+3x-1}{x}\right]$ (3)

6.3 Determine the average gradient of $f(x) = x^2 - 1$ between the points, where $x = 2$ and $x = 6$. (3)

6.4 Determine the equation of a tangent to the curve defined by $g(x) = 5x^2 - 3x$ at the point where $x = 1$. (5)

[19]

QUESTION 7

Given: $f(x) = x^3 - 2x^2 - 4x + 8$

7.1 Show that $(x - 2)$ is a factor of the function $f(x)$. (1)

7.2 Determine the y-intercept of f. (1)

7.3 Determine the x-intercepts of f. (3)

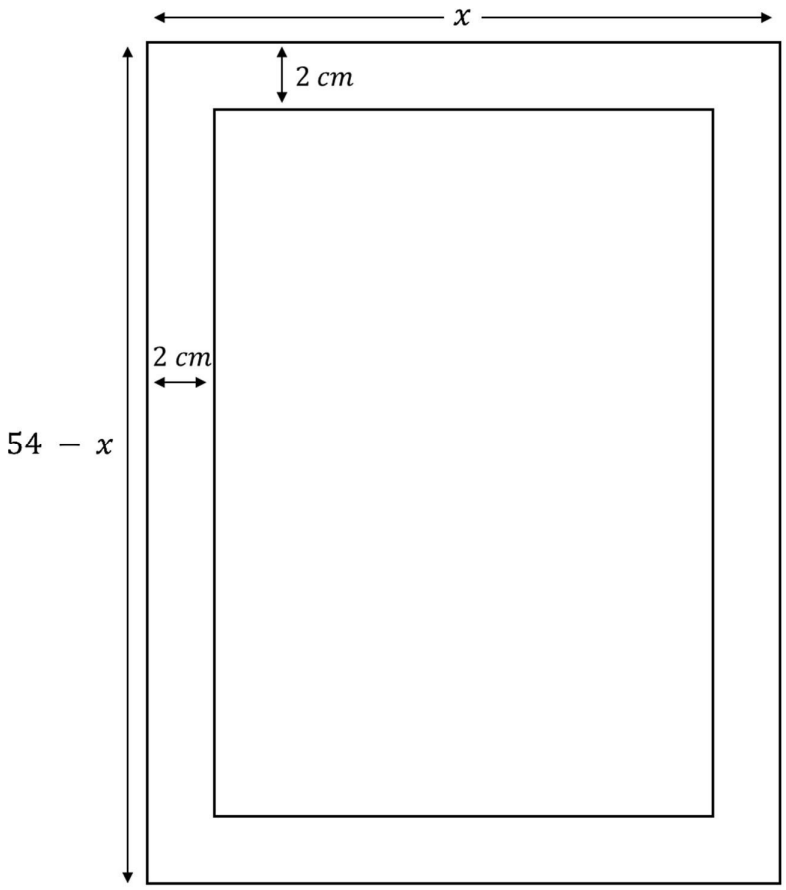
7.4 Determine the coordinates of the turning points of f. (5)

7.5 Sketch the graph of f on the same set of axes on the ANSWER SHEET provided. Clearly show the turning points and the intercepts with the axes. (3)

[13]

QUESTION 8

The diagram below shows a sketch of a rectangular mirror inside a rectangular wooden frame. The wooden frame is 2 cm wide. The outer measurements of the frame are $(54 - x)$ cm in length and x cm in width.



8.1 Express the length and the breadth of the mirror in terms of x. (2)

8.2 Show that the area of the mirror can be written as $Area = -x^2 + 54x - 200$. (2)

8.3 Calculate the maximum area of the mirror. (4)
[8]

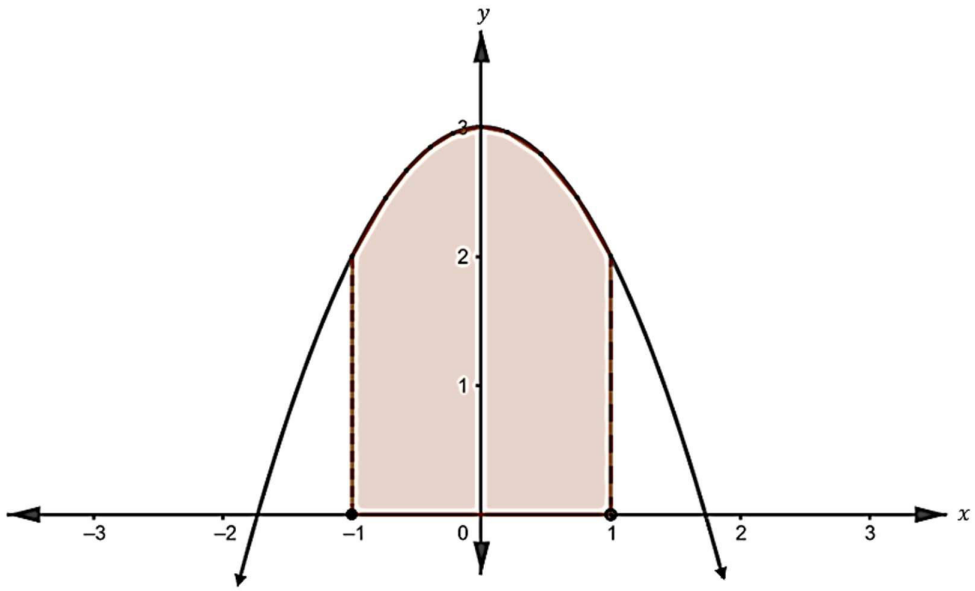
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $4\int(x^{-1} - 3^{2x} - 2)dx$ (3)

9.1.2 $\int(1 - 2x)^2 dx$ (4)

9.2 The curve of the function defined by $f(x) = -x^2 + 3$ is drawn below.



Determine:

9.2.1 $\int(-x^2 + 3)dx$ (2)

9.2.2 The shaded area bounded by the curve and the x-axis, between the points where $x = -1$ and $x = 1$ (4)

[13]

TOTAL: 150

END

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = log_a\, b,$ $\quad a > 0, a \neq 1$ and $b > 0$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \qquad \int kx^n dx = k.\frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, x > 0 \qquad \int \frac{k}{x} dx = k \ln x + C, x > 0$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + C, a > 0 \qquad \int ka^{nx} dx = k.\frac{a^{nx}}{n \ln a} + C, a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad tan\,\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\quad \frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \qquad a^2 = b^2 + c^2 - 2bc.cosA$

Area of $\Delta ABC = \frac{1}{2} ab.sinC$

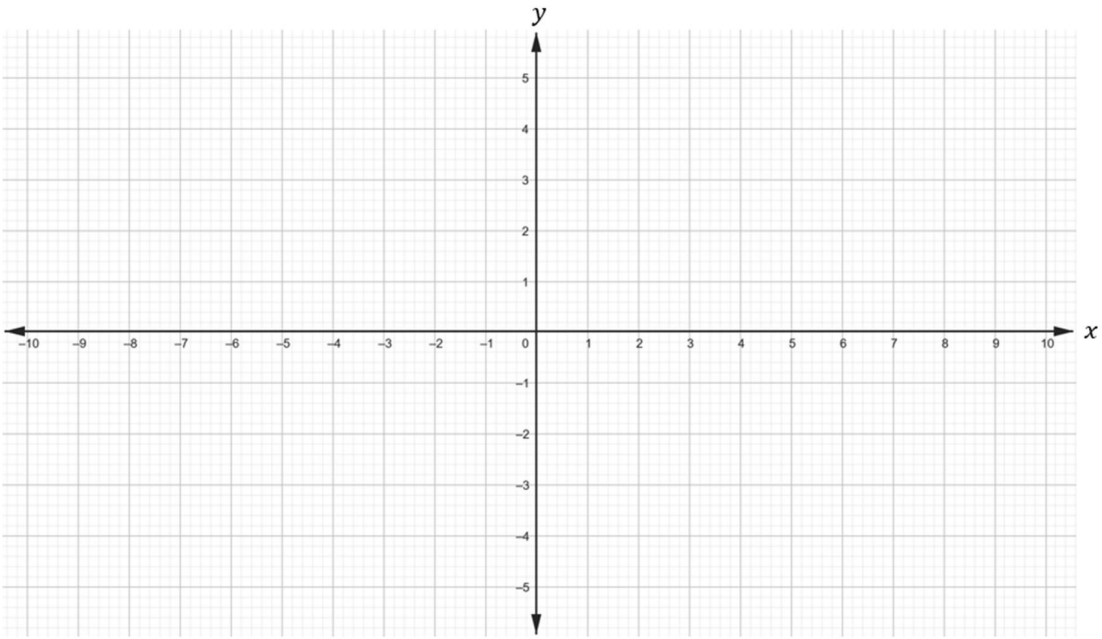
$$sin^2\theta + cos^2\theta = 1 \qquad 1 + tan^2\theta = sec^2\theta \qquad 1 + cot^2\theta = cosec^2\theta$$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n$	where n = rotation frequency
Angular velocity $= \omega = 360°n$	where n = rotation frequency
Circumferential velocity $= v = \pi Dn$	where D = diameter and n = rotation frequency
Circumferential velocity = $v = \omega r$	where ω = angular velocity and r = radius
Arc length $= s = r\theta$	where r = radius and θ = central angle in radians
Area of a sector $= \frac{rs}{2}$	where r = radius, s = arc length
Area of a sector $= \frac{r^2\theta}{2}$	where r = radius and θ = central angle in radians
$4h^2 - 4dh + x^2 = 0$	where h = height of segment, d = diameter of circle and x = length of chord
$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$	where a = width the equal parts, $m_1 = \frac{o_1+o_2}{2}$ $O_n = n^{th}$ ordinate and n = number of ordinates
	OR
$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$	where a = width of the equal parts, $o_n = n^{th}$ ordinate and n =number of ordinates

CANDIDATE'S NAME: _____

QUESTION 4.1.3



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QUESTION 7.5