GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2025

MATHEMATICS

(PAPER 1)

MATHEMATICS P1

C2611E

TIME: 3 hours

MARKS: 150

9 pages + an information sheet

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round-off your answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 4x = 0$ (2)

1.1.2 $2x^2 - \frac{1}{2} = 3x$ (correct to TWO decimal places) (4)

1.1.3 $3x^2 + 5x \geq 2$ (4)

1.1.4 $2^{2x} + 2^x - 6 = 0$ (3)

1.1.5 $x^2 - 2x + 3 + \frac{2}{x^2 - 2x} = 0$ (4)

1.1.6 $\sqrt{x + 5} - x = -1$ (4)

1.2 Solve for x and y simultaneously:

$x + 2y = 5$ and $2y^2 - xy - 4x^2 = 8$ (6)

1.3 For which values of k will the roots of $6x^2 + 6 = 4kx$ be real and equal? (3)

[30]

QUESTION 2

2.1 The first three terms of an arithmetic pattern are:

$-5\,;\ 2\,;9\,;\ldots$

2.1.1 Write down the next two terms of the pattern. (2)

2.1.2 Show that the sum of the first n terms of the pattern is given by:

$S_n = \frac{1}{2}\text{n}(7\text{n} - 17)$ (3)

2.2 The first four terms of a quadratic pattern are:

$x\,;\ 3x - 5\,;4x - 3\,;\ 5x + 1\,;\ \ldots$

2.2.1 Determine the value of x. (3)

2.2.2 If the pattern continues indefinitely, prove that all the terms of the pattern are positive. (5)

2.3 Consider the geometric series:

$$\frac{1}{2}(p-3)+\frac{3}{4}(p-3)^2+\frac{9}{8}(p-3)^3+\cdots;\text{ for } p\neq 3$$

2.3.1 Determine the values of p for which the series converges. (4)

2.3.2 If the sum to infinity of the series is 1, determine the value of p. (3)

[20]

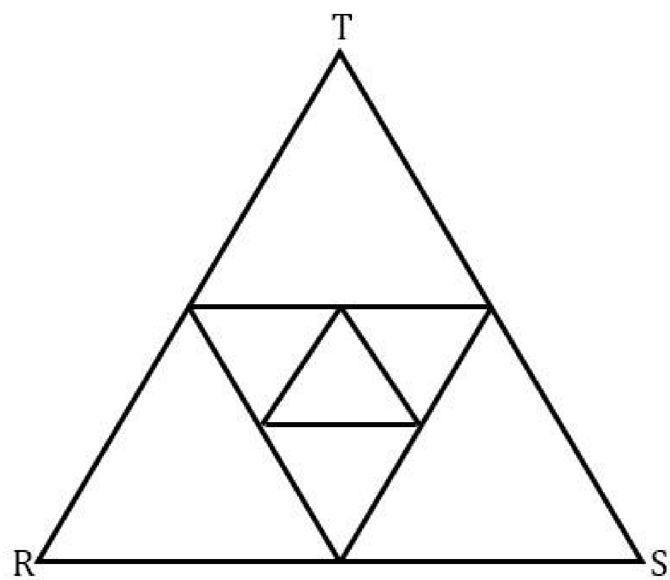
QUESTION 3

3.1 If

$$\sum_{k=2}^{n} 2(3^{k-1}) = 59\,046,\ \text{determine the value of } n.$$

(5)

3.2 An equilateral triangle RST with sides of length $12p$ units is drawn. A second triangle is drawn by joining the midpoints of the sides of the first triangle RST. Each triangle thereafter is drawn by joining the midpoints of the sides of the previous triangle as shown on the sketch, and this continues indefinitely.



3.2.1 Write down, in terms of p, the length of each side of the second triangle. (1)

3.2.2 Calculate, in terms of p, the perpendicular height of ΔRST. (2)

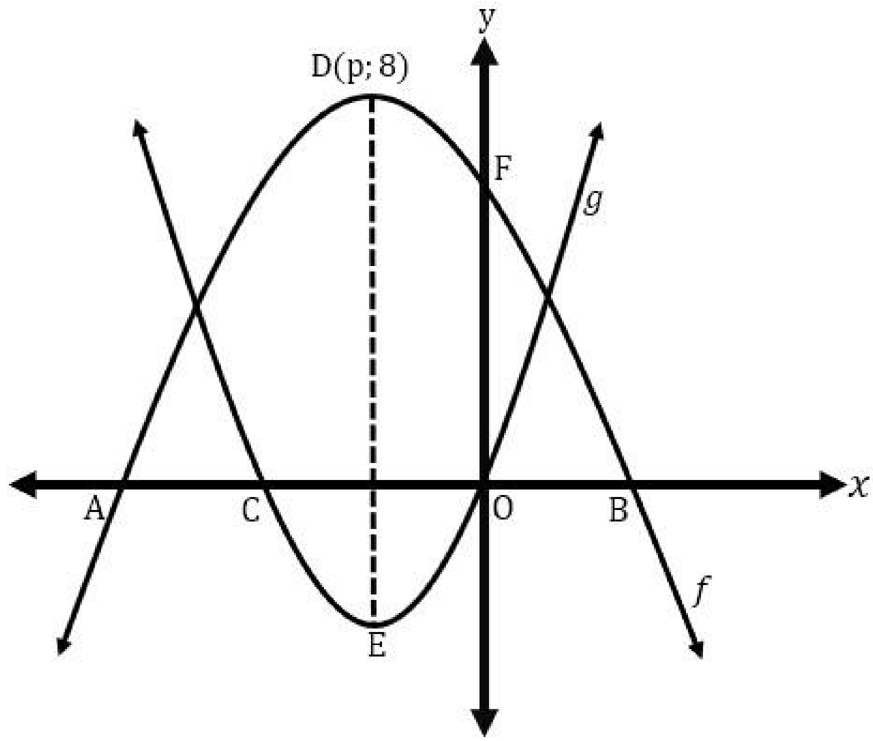
3.2.3 Show that the sum of the areas of all the triangles formed will not exceed $48\sqrt{3}p^2$. (5)

[13]

QUESTION 4

The graphs of $f(x) = ax^2 + bx + \frac{15}{2}$ and $g(x) = x^2 + 2x$ are sketched below.

A and B are the x-intercepts of graph f. D(p ; 8) is the turning point of graph f. C is the x-intercept of graph g. Graph g passes through the origin. E is the turning point of graph g.



4.1 Write down the coordinates of point F. (1)

4.2 Determine the coordinates of C. (2)

4.3 The turning points of graphs f and g lie on the same vertical line DE.

Determine:

4.3.1 The value of p (1)

4.3.2 The length of DE (2)

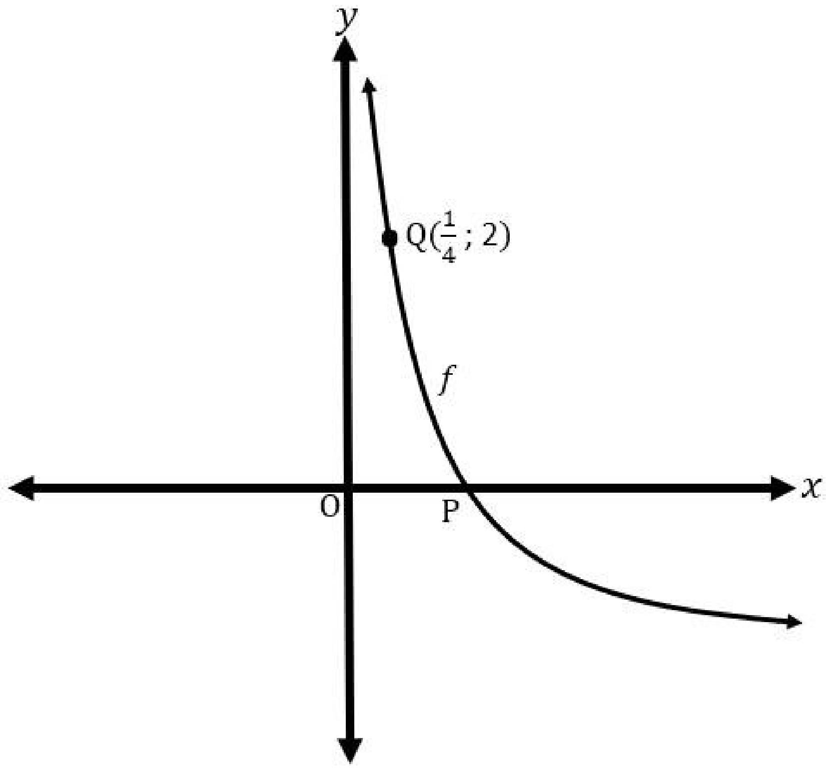
4.4 Show that $a = -\frac{1}{2}$ and $b = -1$. (4)

4.5 Determine the equation of a straight line joining the points of intersection of graphs f and g. **(Round-off your answers to 2 decimal places.)** (5)

[15]

QUESTION 5

The graph of $f(x) = \log_a x$ is sketched below. $Q(\frac{1}{4}; 2)$ is a point on the graph of f. P is the point of intersection of graph f and the x-axis.



5.1 Determine:

5.1.1 The value of a (2)

5.1.2 The inverse of graph f and write your answer in the form $y = \cdots$ (2)

5.2 Sketch the graph of f^{-1} showing intercept(s) with axes and at least one other point on the graph. (3)

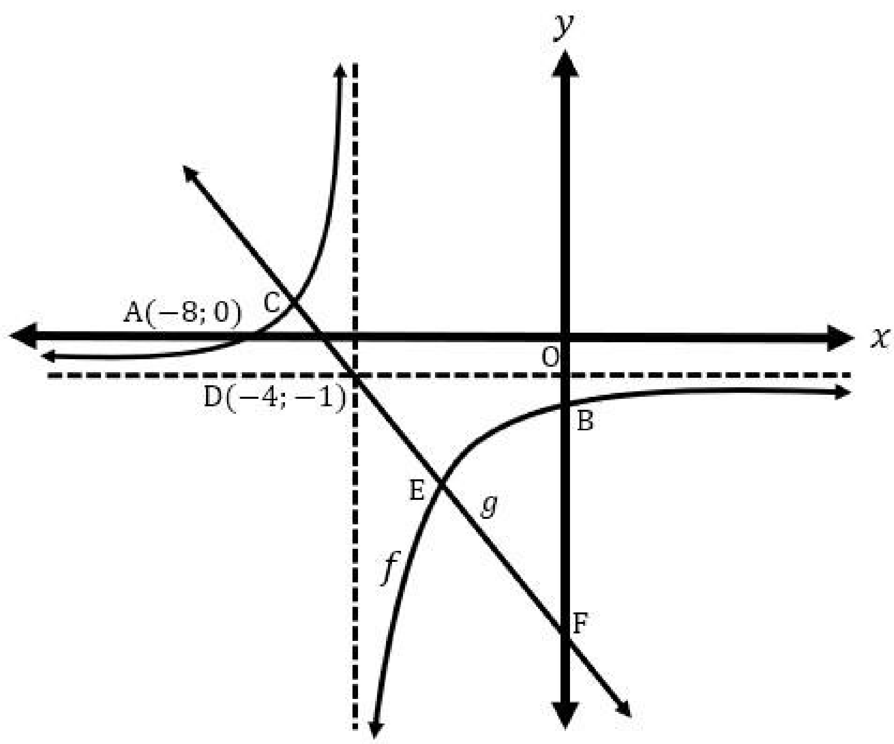
5.3 Determine the values of x for which $f(x) > -5$. (3)

[10]

QUESTION 6

The sketch below shows the graphs of $f(x) = \frac{a}{x+p} + q$ and $g(x) = -x + k$.

The graph $f(x)$ cuts the x-axis at A$(-8\,;0)$ and cuts the y-axis at B. The asymptotes of graph f intersect at point D$(-4\,;-1)$. Graph g passes through point D, meets graph f at points C and E, and then cuts the y-axis at point F, as shown on the sketch.



6.1 Determine:

6.1.1 The value of k. (2)

6.1.2 The values of a, p and q. (4)

6.2 Determine the values of x for which $f(x) \geq g(x)$. (5)

6.3 A graph represented by $h(x) = x + t$ is drawn on the same set of axes as f and g.
For which values of t will graph h be a tangent to graph f? (6)
[17]

QUESTION 7

7.1 Given: $f(x) = \frac{3}{x}$

Determine $f^{/}(x)$ from first principles. (5)

7.2 Determine:

7.2.1 $D_x\left[\frac{\sqrt[3]{x^2} - x^{-\frac{3}{2}}}{\sqrt{x}}\right]$ (4)

7.2.2 $\frac{dy}{dx}$ if $xy - y = x^2 - 1$ (3)

7.3 The gradient of the tangent to the function represented by $f(x) = ax^3 + bx^2$ at the point (1 ; 5) is 12.

7.3.1 Show that $a = 2$ and $b = 3$. (5)

7.3.2 Calculate the coordinates of the points on the curve where the tangent to the curve is parallel to the x-axis. (5)

[22]

QUESTION 8

8.1 The following information is given relating to a cubic graph f:

- $f(0) = 0$ $\quad$ $f(3) = 0$
- $f^{/}(3) = 0$ $\quad$ $f^{/}(1) = 0$
- $f^{//}(2) = 0$ $\quad$ $f^{//}(x) < 0$ for $x > 2$

8.1.1 Explain the meaning of $f^{//}(x) < 0$ for $x > 2$. (2)

8.1.2 Draw a neat sketch of graph f showing all relevant points. (4)

8.1.3 Determine the values of x for which $f^{/}(x).f(x) < 0$. (3)

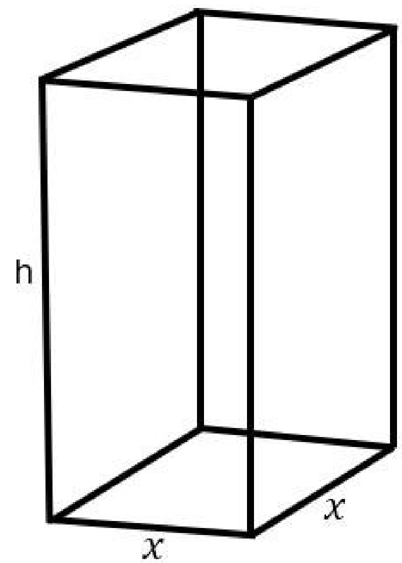
8.2 It is further given that graph f passes through point W$(5\,;\,-40)$.
Determine the equation of graph f and leave your answer in the form $f(x) = px^3 + qx^2 + rx$ where p, q and r are constant values. (4)

[13]

QUESTION 9

An industrial chemical is stored in rectangular containers with a square base of x centimetres. The height of each container is h centimetres as shown on the figure below.

The volume of each container is 2 160 000 cm^3.



9.1 Determine the height of the container in terms of x. (2)

9.2 For safety reasons during the transportation of the containers, a triple layer of material is needed at the base of the container.

(Ignore the thickness of the material.)

Show that the total surface area of the material used for the container is given by:

$$A(x) = 4x^2 + \frac{8\ 640\ 000}{x}$$ (3)

9.3 Determine the dimensions of the container that will ensure that the minimum amount of material is used when making the containers. (5)

[10]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$