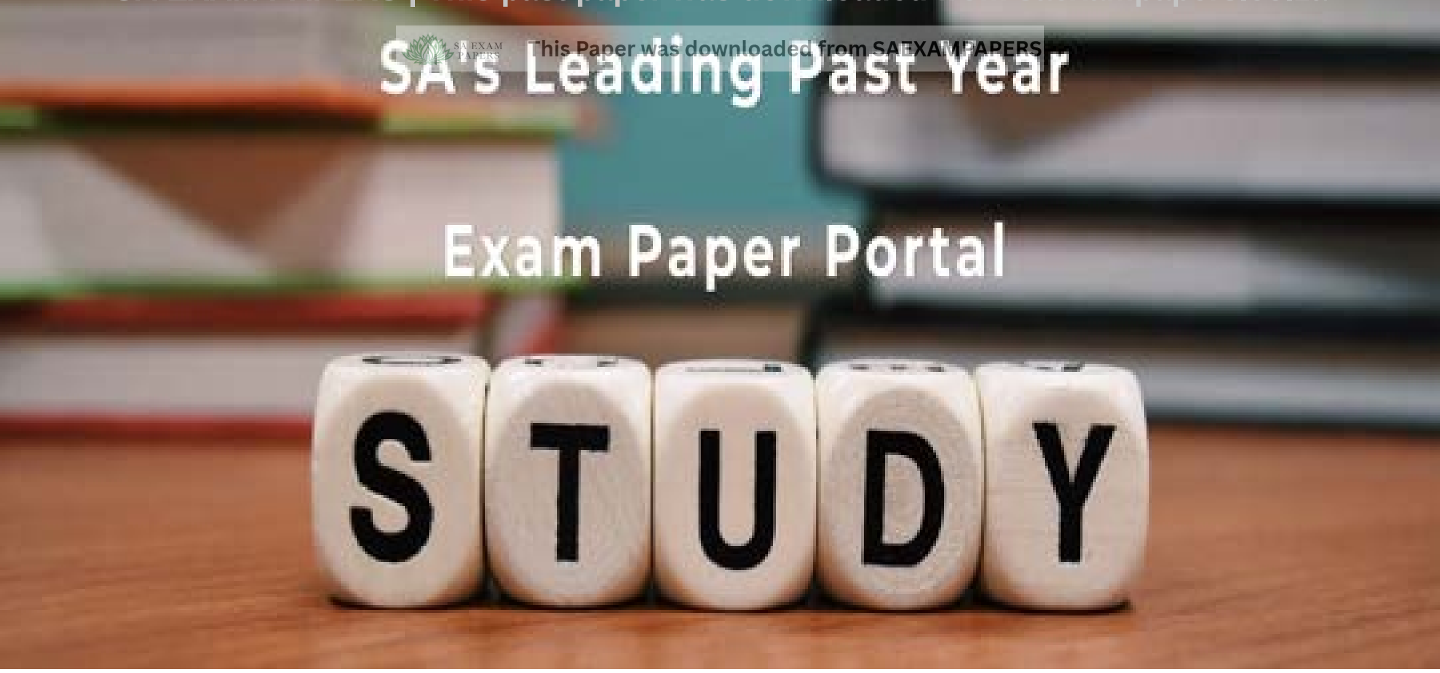
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Exam Paper Portal
STUDY

DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

PROVINSIALE VOORBEREIDENDE EKSAMEN/
PROVINCIAL PREPARATORY EXAMINATION

GRAAD 12/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL 2/PAPER 2

SEPTEMBER 2025

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

Hierdie vraestel bestaan uit 13 bladsye, 'n inligtingsblad en 'n 23 bladsy- SPESIALE ANTWOORDEBOEK./
This question paper consists of 13 pages, an information sheet and a 23-page SPECIAL ANSWER BOOK.

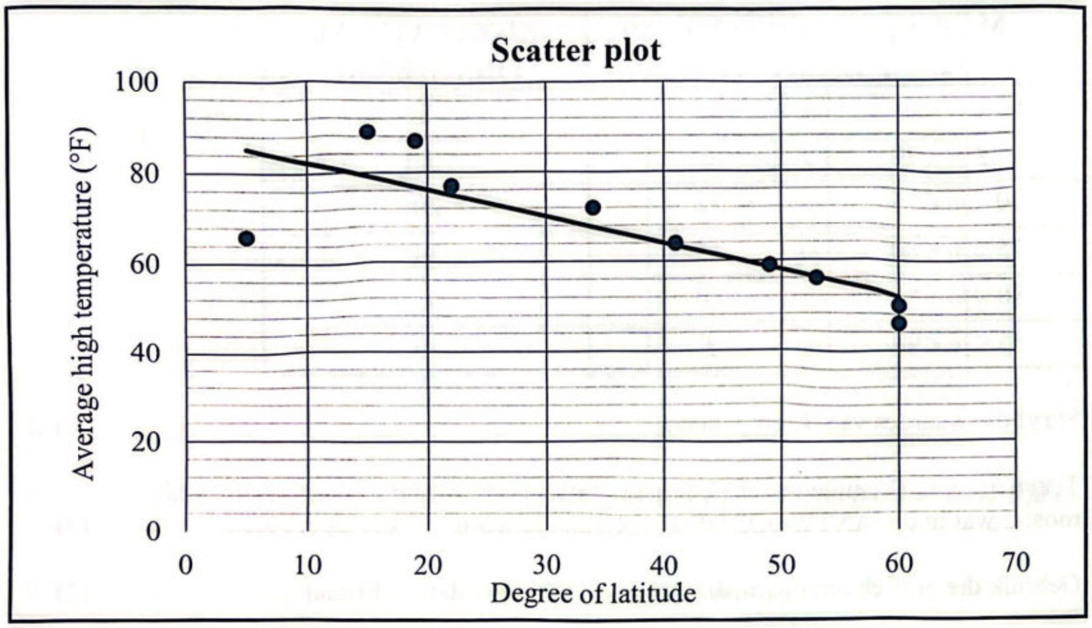
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The scatter plot below shows the degrees of latitude of 10 cities in the Northern Hemisphere and their average high temperatures (in °F). The least squares regression line is also shown.



1.1 Use the scatter plot to describe the strength of the correlation between the latitude of a city and its average high temperature. Motivate your answer. (2)

The table below shows the data that was used to draw the scatter plot above.

Latitude in degrees	5	19	34	53	22	41	60	15	60	49
Average high temperature (in °F)	66	87	72	56	77	64	46	89	50	59

1.2 Determine the equation of the least squares regression line for the data. (3)

1.3 Predict the average high temperature of a city with a latitude of 28 degrees. (2)

1.4 Write down the standard deviation of the average high temperature of the 10 cities. (1)

1.5 Determine the number of cities of which the average high temperature is greater than one standard deviation above the mean. (3)

[11]

QUESTION 2

Fifty boys were weighed and their mass recorded to the nearest kilogram. The results are shown in the table below.

MASS (kg)	FREQUENCY	CUMULATIVE FREQUENCY
$60 \leq m < 65$	2	2
$65 \leq m < 70$	6	8
$70 \leq m < 75$	12	20
$75 \leq m < 80$	14	34
$80 \leq m < 85$	10	k
$85 \leq m < 90$	f	50

2.1 Write down the values of k and f. (2)

2.2 Draw an ogive (cumulative frequency graph) for the data on the grid provided in the ANSWER BOOK. (3)

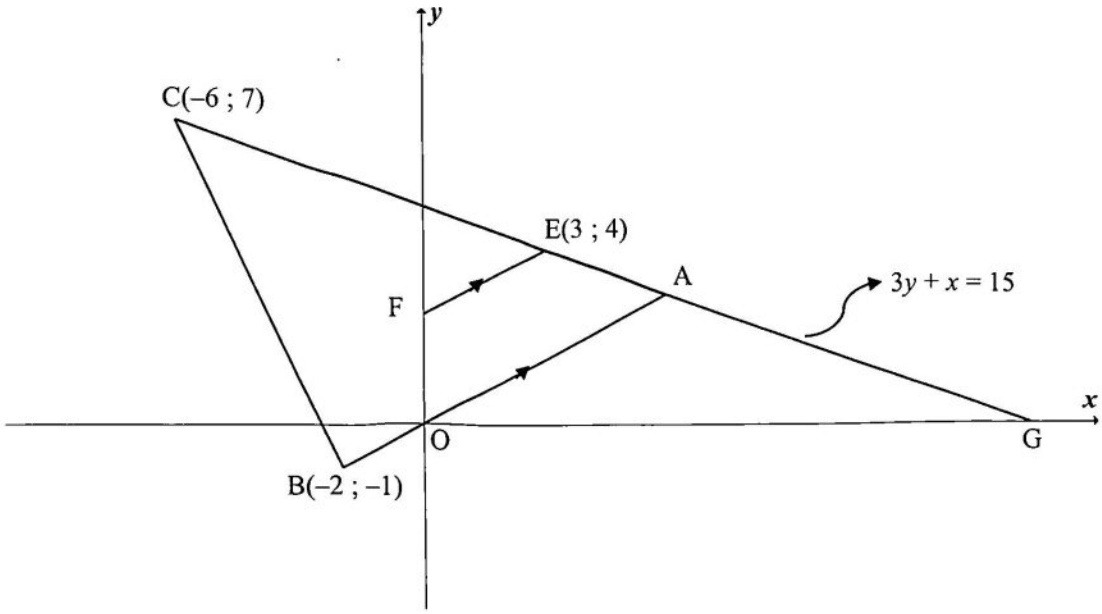
2.3 Use the graph to determine the median mass for this data. (2)

2.4 What percentage of the boys weighed 83 kg or more? (2)

[9]

QUESTION 3

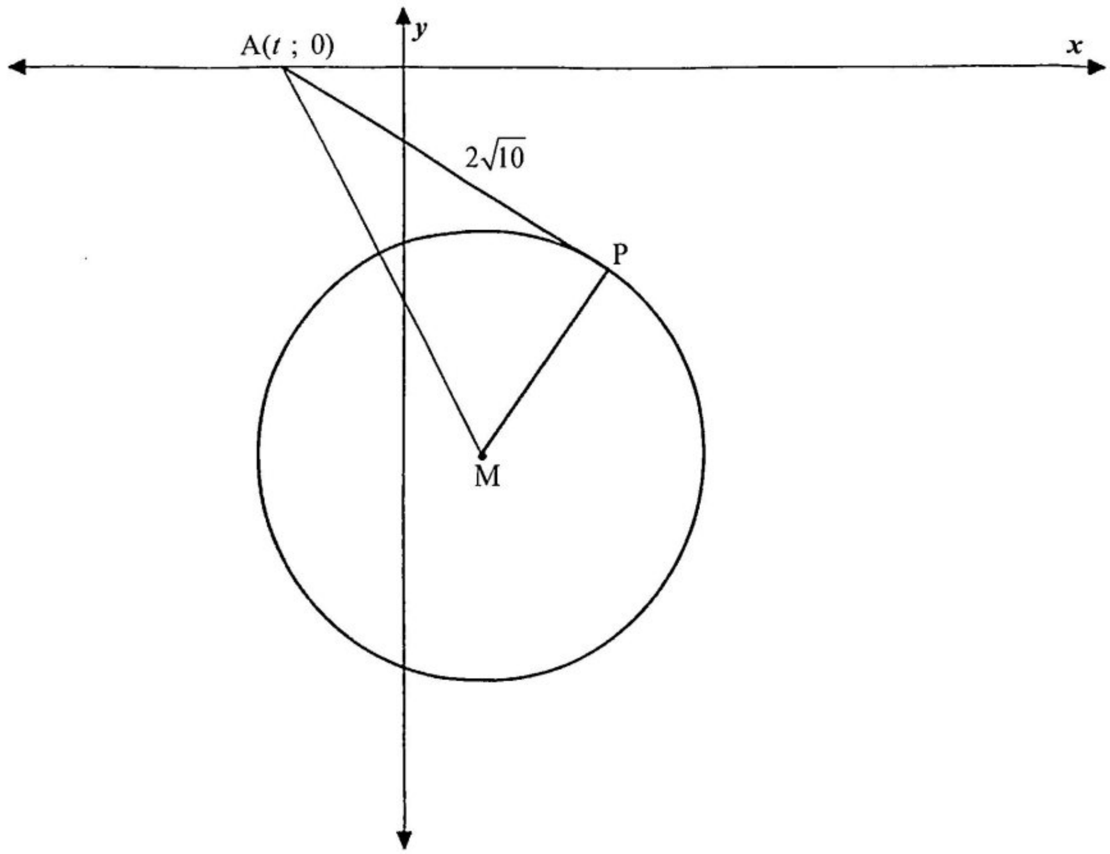
In the diagram below, A, B(–2 ; –1) and C(–6 ; 7) are vertices of ΔABC. CA is produced to cut the x-axis at G. F is a point on the y-axis and E(3 ; 4) a point on CA such that FE || BA. Line AB passes through the origin. The equation of CG is $3y + x = 15$.



3.1	Calculate the gradient of AB.	(2)
3.2	Prove that CB ⊥ BA.	(3)
3.3	Determine the equation of FE in the form $y = mx + c$.	(3)
3.4	Calculate the:	
	3.4.1 Size of $O\hat{F}E$	(3)
	3.4.2 Area of quadrilateral OFEG	(6)
3.5	A(6 ; 3), B, C and D form a rectangle with point D in the first quadrant.	
	3.5.1 Calculate the coordinates of D.	(2)
	3.5.2 A circle is drawn through points A, B and C.	
	(a) Calculate the coordinates of the centre of the circle.	(2)
	(b) Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.	(3)
		[24]

QUESTION 4

In the diagram below, a circle, centred at M, with the equation $x^2 - 4x + y^2 + 10y + 19 = 0$, is drawn. From $A(t\ ;\ 0)$, a tangent is drawn to touch the circle at P. ΔAPM is drawn and $AP = 2\sqrt{10}$.



4.1 Give a reason why $A\hat{P}M = 90°$. (1)

4.2 Calculate the:

4.2.1 Coordinates of M (3)

4.2.2 Length of PM, the radius of the circle (1)

4.3 Show that $t = -3$. (4)

4.4 Another circle, centred at N, with the equation $(x-5)^2 + (y-e)^2 = 40$, is drawn. Centre N lies on produced line MA, with the equation $y = -x - 3$.

4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine whether the two circles, centred at M and N, will intersect, touch externally, or do not intersect at all. (5)

[16]

QUESTION 5

5.1 Given: $\sqrt{13}\sin\alpha - 2 = 0$ and $x \in [90° ; 270°]$

Determine, without the use of a calculator, the value of:

5.1.1 $\tan\alpha$ (4)

5.1.2 $\sin(90° - \alpha)$ (2)

5.2 Consider: $\dfrac{\sin(180° + \theta).\cos(90° + \theta)}{\tan\theta.\cos(-\theta)}$

Simplify the expression to a single trigonometric ratio. (5)

5.3 Determine the general solution of x in the equation $\sin x = 1 - \cos 2x$. (6)

5.4 Consider the following identity: $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$

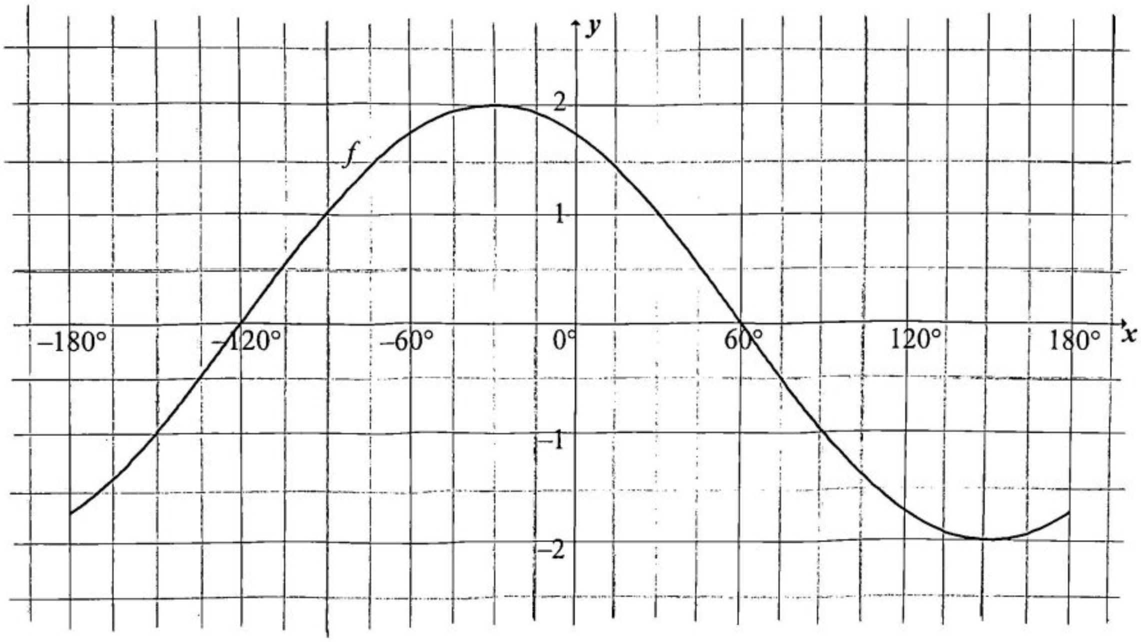
5.4.1 Prove the identity. (2)

5.4.2 Hence, or otherwise, show, without using a calculator, that $\dfrac{(\sin 7x + \sin 3x)}{(\cos 7x + \cos 3x)} = \tan 5x$ (5)

[24]

QUESTION 6

In the diagram below, the graph of $f(x) = a\cos(x+b)$ is drawn for the interval $x \in [-180°\,;180°]$.

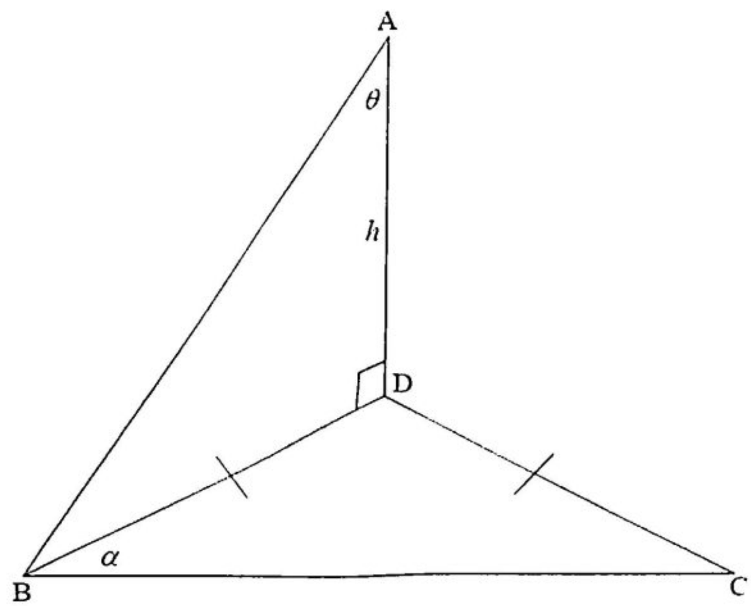


6.1	Use the graph to determine the values of a and b.	(2)
6.2	Draw the graph of $g(x) = \sin 2x + 1$ for the interval $x \in [-180°\,;180°]$ on the grid provided in the ANSWER BOOK. Clearly show the intercepts with the axes as well as the coordinates of the turning points.	(3)
6.3	Write down the period of g.	(1)
6.4	Determine the range of $2g(x)$.	(3)
6.5	Use the graphs to determine the value(s) of x for which:	
	6.5.1 $f(x) < g(x)$, in the interval $x \in [-180°\,;0°]$	(2)
	6.5.2 $\tan(x+b)$ is undefined in the interval $x \in [-180°\,;180°]$	(2)
6.6	Graph of g is shifted 45° to the left to obtain a new graph p. Determine the equation of p in its simplest form.	(2)
		[15]

QUESTION 7

In the diagram, AD is a vertical pole such that $AD \perp BD$. B, D and C lie in the same horizontal plane. The pole is anchored at B.

$B\hat{A}D = \theta$, $D\hat{B}C = \alpha$, $AD = h$ and $DB = DC$.

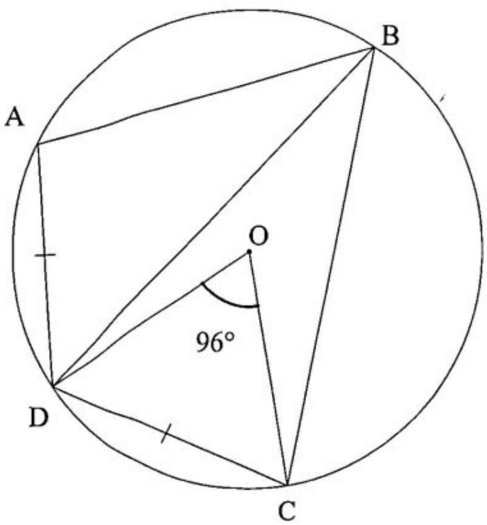


7.1	Determine BD in terms of h.	(2)
7.2	Hence, show that $BC = 2h\tan\theta.\cos\alpha$.	(4)
7.3	If $\theta = 40°$, $\alpha = 25°$ and $h = 5$ metres, determine the area of ΔBDC.	(3)
		[9]

Provide reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram, O is the centre of the circle. Cyclic quadrilateral ABCD and chord BD are drawn such that AD = DC. $D\hat{O}C = 96°$.



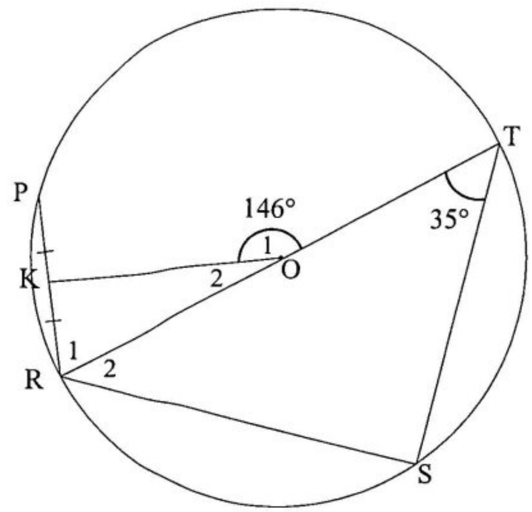
Calculate, with reasons, the size of:

8.1.1 $D\hat{B}C$ (2)

8.1.2 $A\hat{B}D$ (2)

8.1.3 $A\hat{D}O$ (5)

8.2 In the diagram, O is the centre of the circle. P, R, S and T are points on the circle. ΔRTS is drawn. OK bisects chord PR at K. $\hat{O}_1 = 146°$ and $\hat{T} = 35°$.

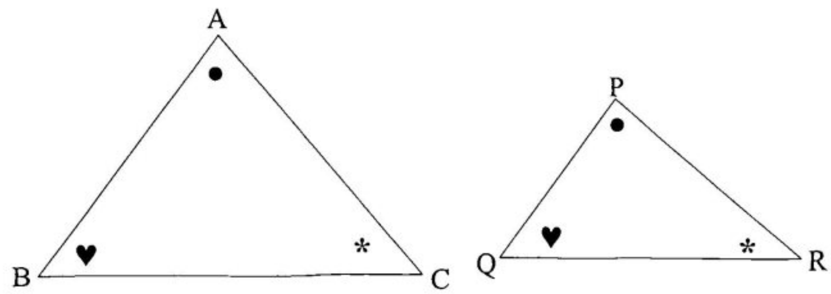


Calculate, with reasons, the size of $P\hat{R}S$. (7)

[16]

QUESTION 9

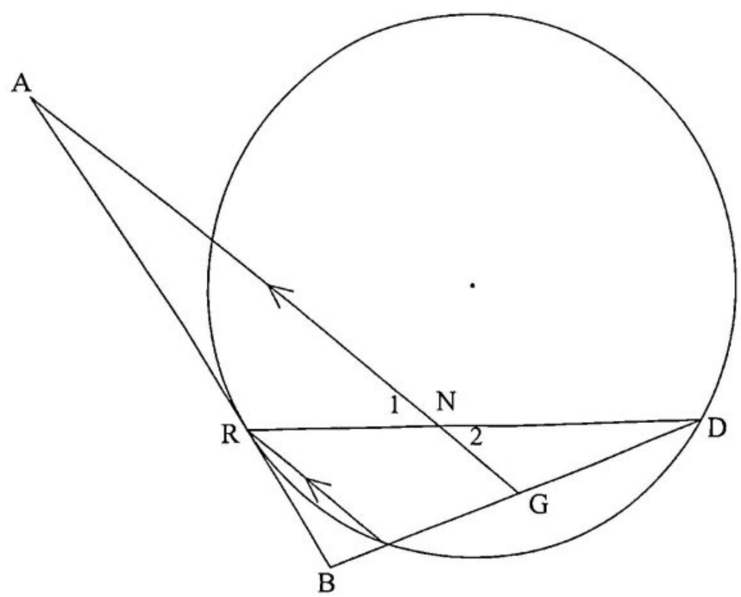
9.1 In the diagram below, ΔABC and ΔPQR are drawn such that $\hat{A} = \hat{P}$; $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion,

i.e. $\frac{AB}{PQ} = \frac{AC}{PR}$ (6)

9.2 In the diagram below, AB is a tangent to the circle at R. C and D are points on the circle. ΔRCB and ΔRCD are drawn. G is a point on chord CD such that AG || RC. Chord RD and AG intersect at N.



Prove, with reasons, that:

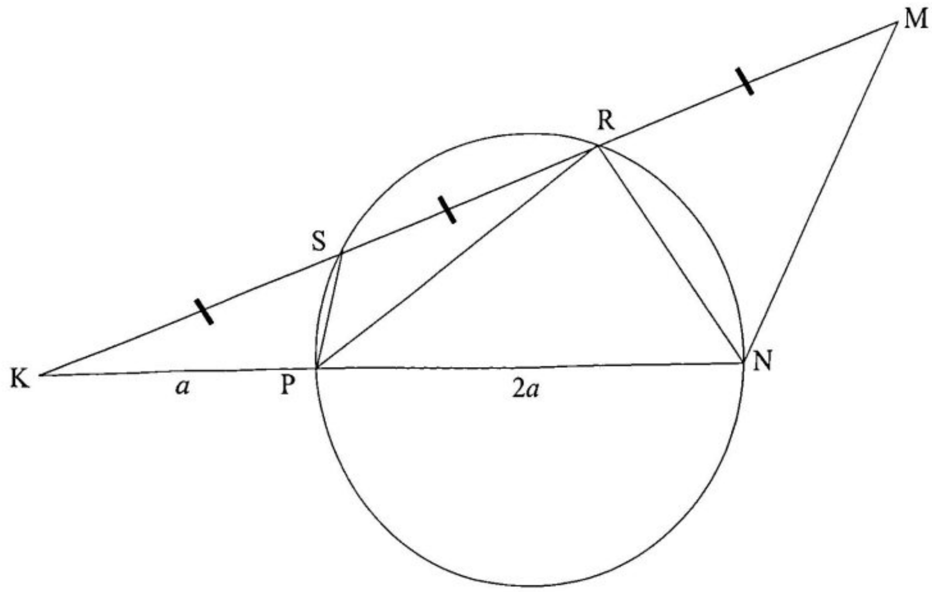
9.2.1 ΔDNG ||| ΔANR (5)

9.2.2 $B\hat{R}C = \hat{A}$ (2)

[13]

QUESTION 10

In the diagram, PNRS is a cyclic quadrilateral of the circle. RS and NP are produced to meet at K. NM meets SR produced at M. Chord PR is drawn.
KS = SR = RM, KM = 18 units, KP = a units and PN = $2a$ units



10.1 Prove, with reasons, that:

10.1.1 PS || NM (2)

10.1.2 PN is a tangent to a circle passing through R, N and M, at N (4)

10.1.3 $\frac{NM}{NR} = \frac{1}{4}a$ (4)

10.2 If it is further given that ΔPNR ||| ΔNMR, determine the length of NM in terms of a. (3)
[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = (1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1};\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$