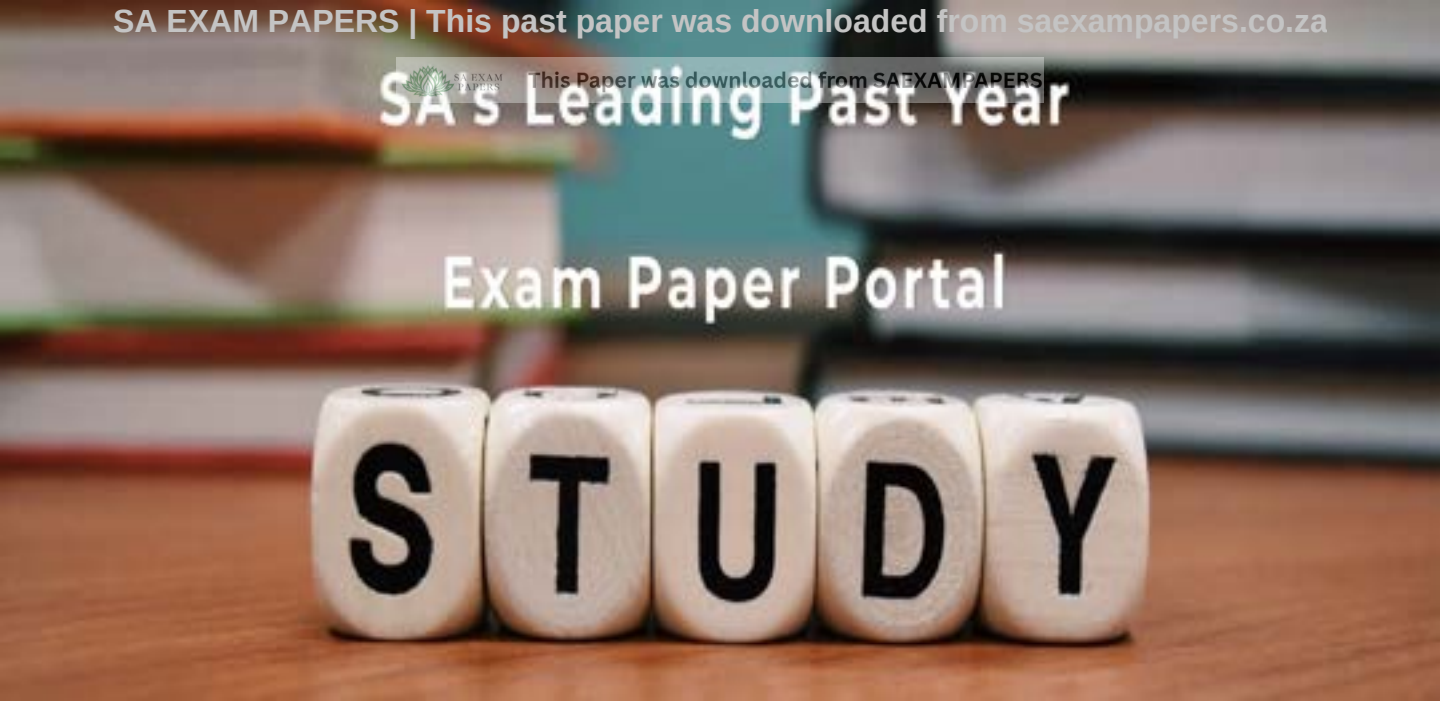


GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2025

TECHNICAL MATHEMATICS

(PAPER 1)

TECHNICAL MATHEMATICS P1

C2091E

TIME: 3 hours

MARKS: 150

X05

9 pages + a 2-page information sheet and 2 answer sheets

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTIONS 3.3.3 and 7.4 on the ANSWER SHEETS provided. Write your name in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(2x+4)=0$ (2)

1.1.2 $5(x^2-3x)=3(x^2+4)-4$ (correct to TWO decimal places) (4)

1.1.3 $x^2-4x\leq 0$ (3)

1.2 Solve for x and y if:

$x+y-6=0$ and $xy=8$ (6)

1.3 The electrical current (I) can be calculated by using the formula $I=\frac{E}{R+r}$ where E represents the electromotor force, r the internal resistance and R the external resistance.

1.3.1 Make r the subject of the formula. (2)

1.3.2 Hence, or otherwise, calculate the internal resistance (r) given that:

- The current (I) is 5 ampere.
- The electromotor force (E) is 13,05 volts.
- The external resistance (R) is 2 ohms. (2)

1.4 Calculate 1101_2-3 and leave your answer in binary form. (2)

[21]

QUESTION 2

2.1 Given: $f(x)=2x^2+x+6$

2.1.1 Determine the numerical value of the discriminant. (2)

2.1.2 Hence, describe the nature of the roots of the equation. (1)

2.2 Determine the numerical value of t for which the equation $-x^2+3x+2t=0$ will have non-real roots (without solving the equation). (4)

[7]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $\dfrac{2^{-2}.\ 5^{0}}{3^{-1}}$ (2)

3.1.2 $(2\,.\sqrt[3]{8a^{2}})^{\frac{3}{2}}$ (3)

3.1.3 $(\log_2 8)(\log_3 32)(\log_{16} 243)\left(\dfrac{1}{5}\right)$ (4)

3.2 Solve for x:

3.2.1 $\log(x+1)-\log x-1=0$ (4)

3.2.2 $3^{2x}+3^{2x+1}=36$ (4)

3.3 Given the complex number: $z=\sqrt{2}(\cos 60°+i\ sin\ 60°)$

3.3.1 Express z in the form $a+bi$. (3)

3.3.2 Write down the conjugate of z. (1)

3.3.3 Represent z on an Argand diagram on the complex plane on the ANSWER SHEET provided. (2)

[23]

QUESTION 4

4.1 Given: The functions f and g defined by $f(x) = -x + 1$ and $g(x) = 2(x + 1)^2 - 8$

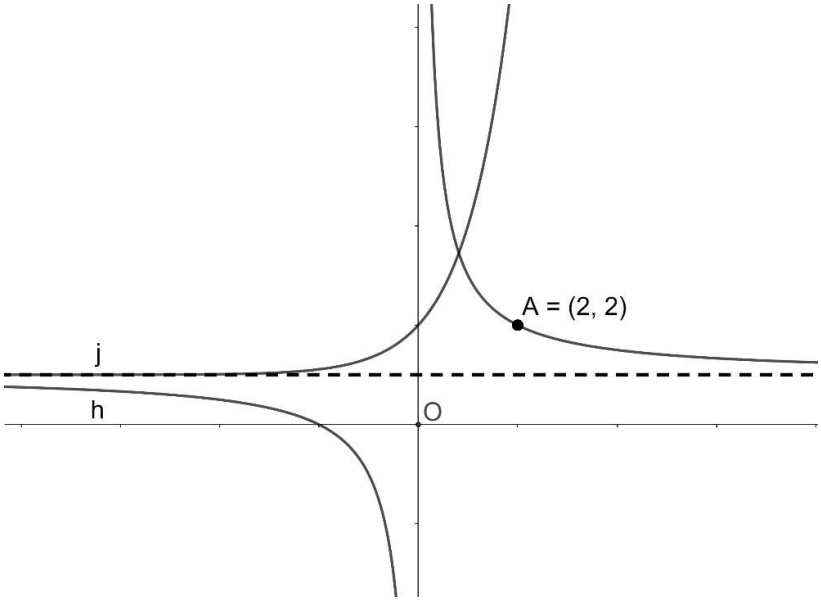
4.1.1 Determine the turning point of g. (2)

4.1.2 Determine the x-intercept(s) of g. (4)

4.1.3 Determine the point(s) of intersection of f and g where $x > 0$. (Show ALL calculations.) (5)

4.1.4 Determine the range of g. (2)

4.2 The graph below represents the functions defined by $h(x) = \frac{a}{x} + q$ and $j(x) = 3^x + 1$. Graph $h(x)$ and $j(x)$ share the same horizontal asymptote and $A(2\ ;\ 2)$ is a point on h.

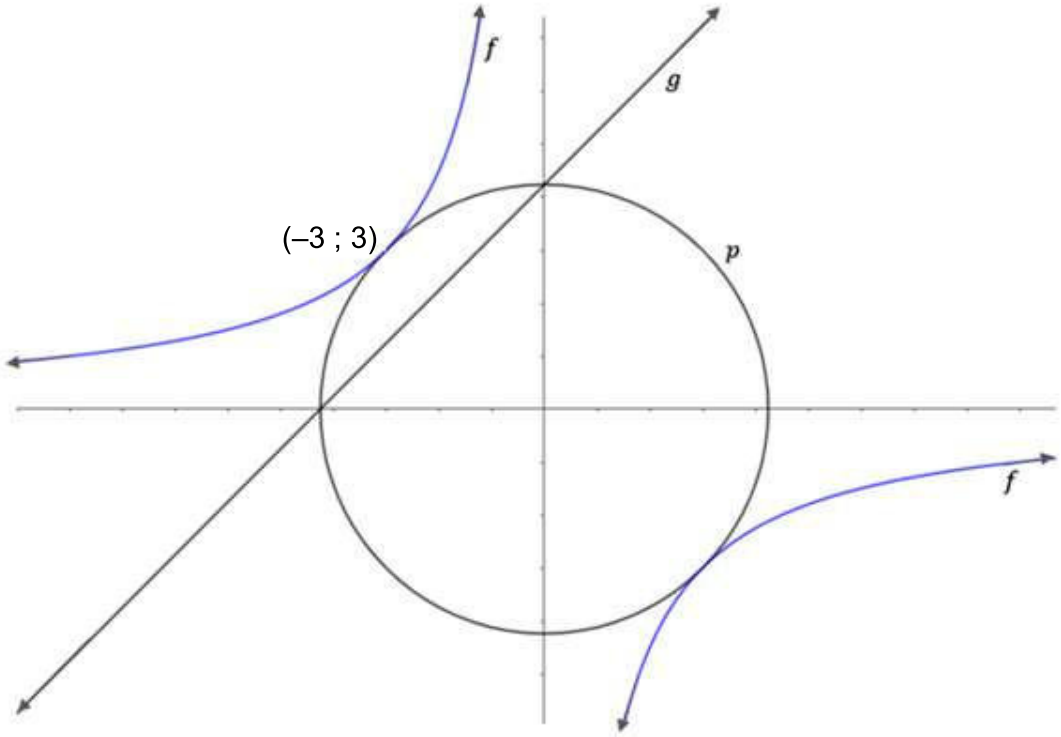


4.2.1 Write down the equation of the asymptote of j. (1)

4.2.2 Hence, determine the equation of h. (4)

4.2.3 Determine the range of h. (1)

4.3 Sketched below are the graphs of $p(x) = \pm\sqrt{r^2 - x^2}$, $g(x) = mx + c$ and $f(x) = \frac{a}{x}$. $f(x)$ and $p(x)$ share point $(-3\ ;\ 3)$.



Determine the:

4.3.1 Equation of f (3)

4.3.2 Equation of p (3)

4.3.3 Equation of g (3)

4.3.4 Equation of the asymptotes of f (2)

4.3.5 Domain of p (2)

[32]

QUESTION 5

5.1 Steam in a boiler was heated to $150\ °C$ and then allowed to cool. Its temperature T (in °C) was recorded each minute, as shown in the following table.

Time (min)	0	1	2	3	4	5
Temperature (°C)	150,0	142,8	138,5	135,2	132,7	130,8

Determine the rate of depreciation. (Hint: Use the first and last value.) (4)

5.2 The annual effective interest rate charged on an investment is 12,5% compounded quarterly. Calculate the nominal interest rate of the investment. (4)

5.3 Mr Cassim invested R4 800 in a savings account that offers the following:

- An interest rate of 8,15% p.a., compounded monthly
- An amount of R1 200 was withdrawn from the account at the end of the first year

Calculate the total amount of money that will be in the account at the end of the 5th year. (6)

[14]

QUESTION 6

6.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x) = 7 + 5x$. (5)

6.2 Determine $f'(x)$ if $f(x) = 2\sqrt{x} - 3x^{-1} + 4$. (3)

6.3 Given: $y = \frac{t^2-4}{t-2}$

6.3.1 Simplify y. (2)

6.3.2 Determine $\frac{dy}{dx}$. (1)

6.4 A tangent is drawn to the curve of y= $x^2 - 4x$. Determine the x-coordinate of the point of contact (the point where the tangent touches the curve). (5)

[16]

QUESTION 7

Given: $f(x) = (x-2)^2(x+3)$

7.1 Write down the x-intercepts. (2)

7.2 Determine the coordinates of the y-intercept. (3)

7.3 Hence, determine the turning points. (5)

7.4 Sketch the graph of f on the set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes. (4)

[14]

QUESTION 8

A closed rectangular gift box has a volume of 400 cm^3. \The width is 5 cm and the length is x cm.

Formulae:

$V = lbh$

$TSA = 2lb + 2bh + 2lh$

8.1 Write down an expression for the height of the box in terms of x. (2)

8.2 Hence, show that the total surface area of the box can be given as:

$$TSA = 10x + \frac{800}{x} + 160$$ (3)

8.3 Determine the value of x for which the total surface area will be a minimum. (3)

8.4 Calculate the minimum total surface area of the gift box. (2)

[10]

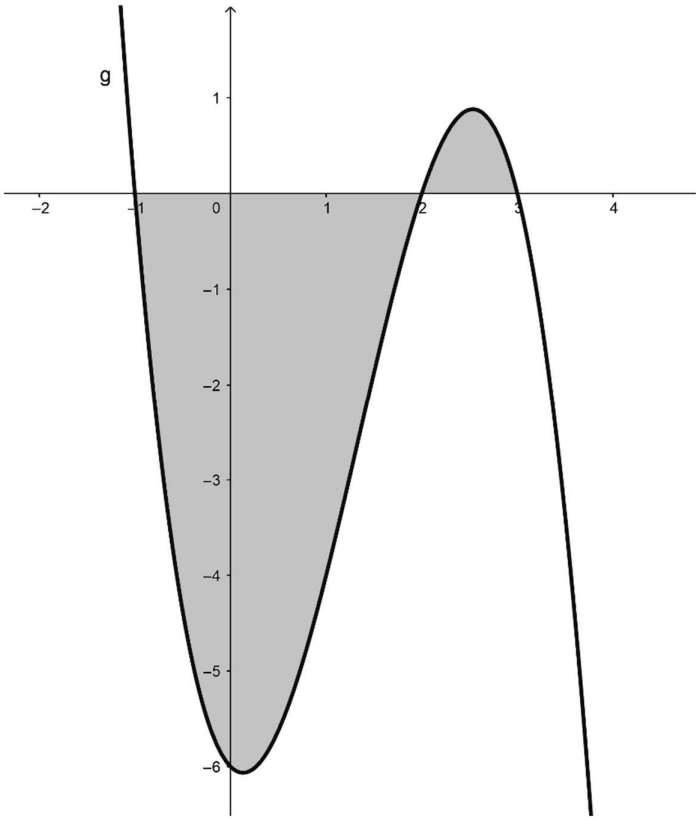
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int \left(-6x^2 + \frac{2}{x} - 3^x\right) dx$ (3)

9.1.2 $\int (x + y)(x - y)\, dx$ (3)

9.2 The sketch below shows the shaded area bounded by function g defined by $g(x) = -x^3 + 4x^2 - x - 6$ and the x-axis between the points where $x = -1$, $x = 2$ and $x = 3$.



Determine the shaded area bounded by function g between the points $x = -1$, $x = 2$ and $x = 3$. Show ALL calculations. (7)

[13]

TOTAL: 150

END

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1+i)^n \qquad A = P(1-i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \ n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \ x > 0 \text{ and } k \in \mathbb{R} \ ; \ k \neq 0$$

$$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \ a > 0 \ ; \ a \neq 1 \text{ and } \ k, a \in \mathbb{R} \ ; \ k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} \ ; \ \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$\pi\ rad = 180°$

Angular velocity $= \omega = 2\ \pi\ n$ where n = rotation frequency

Angular velocity $= \omega = 360°\ n$ where n = rotation frequency

Circumferential velocity $= v = \pi\ D\ n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\ r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4d\,h + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = length of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

$O_n = n^{th}$ ordinate and n = number of ordinates

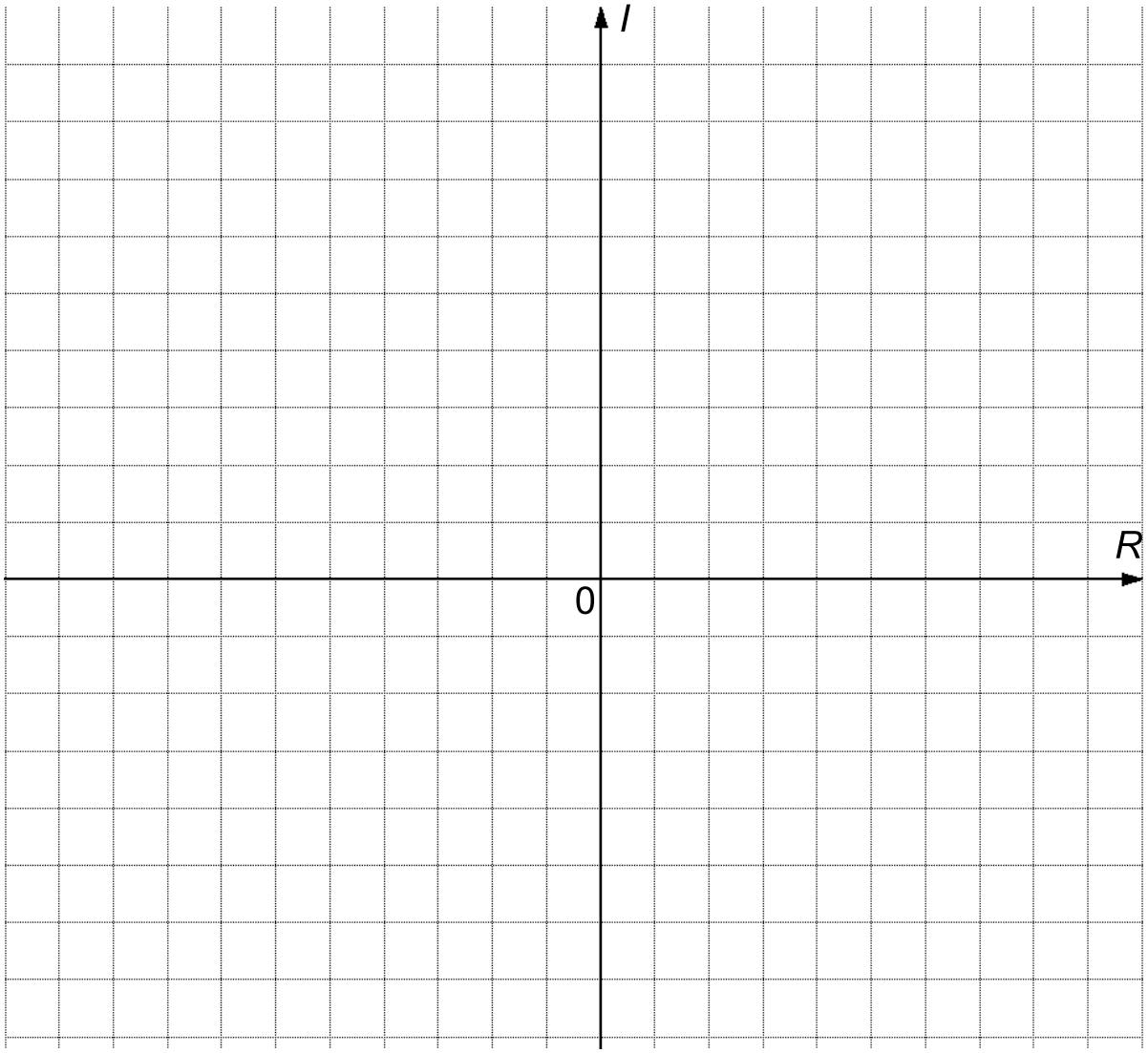
OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = length of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

NAME AND SURNAME:	

QUESTION 3.3.3



ANSWER SHEET

NAME AND SURNAME:	

QUESTION 7.4