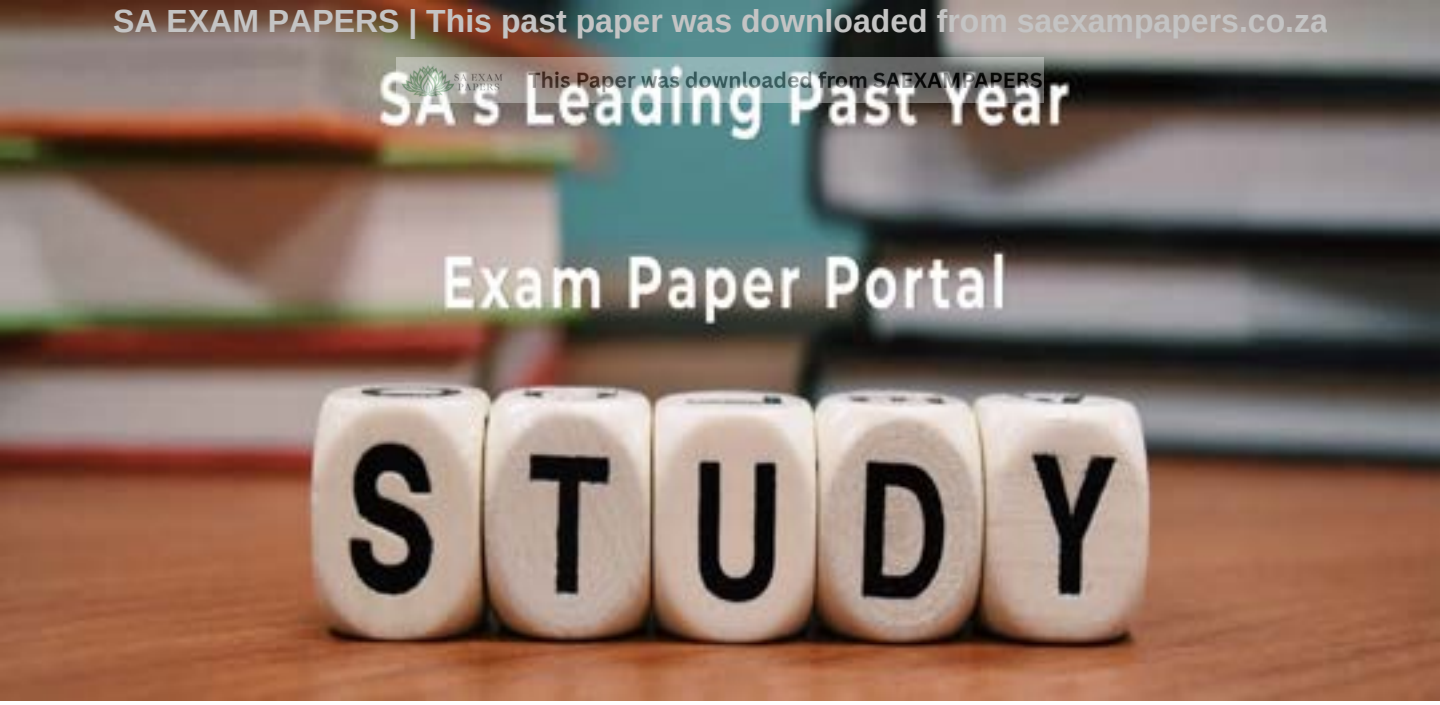


LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

NATIONAL SENIOR CERTIFICATE/*NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE P2*

MARKING GUIDELINES/*NASIENRIGLYNE*

JUNE 2025

MARKS/*PUNTE*: 150

This marking guidelines consists of 17 pages/*Hierdie nasienriglyne bestaan uit 17 bladsye*

NOTE/*NOTA*:

- If a candidate answers a question TWICE, only mark the FIRST attempt/ *As 'n kandidaat 'n vraag twee keer beantwoord, merk slegs die EERSTE poging.*
- Consistent Accuracy applies in all aspects of the marking guidelines/ *Konsekwente akkuraatheid is van toepassing in alle aspekte van die nasienriglyne*

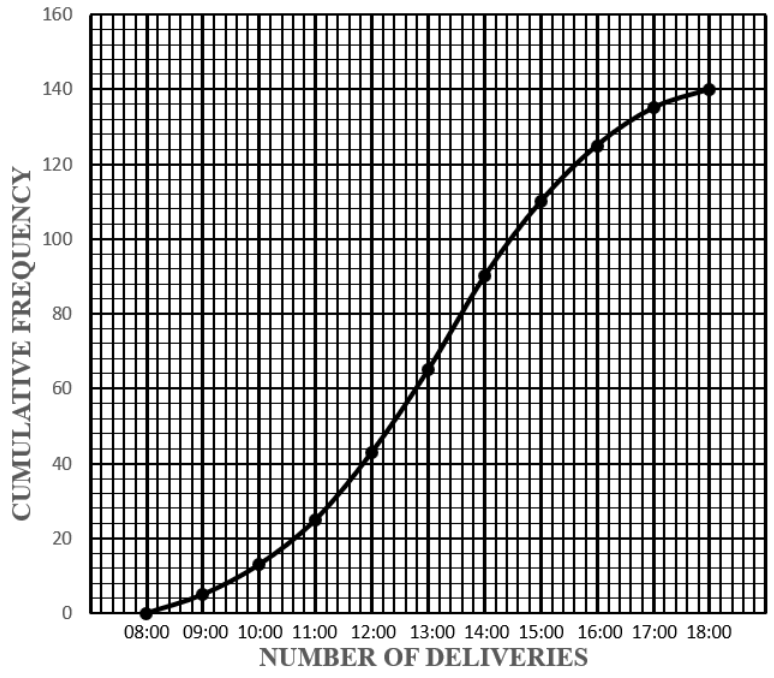
QUESTION/*VRAAG* 1

1.1

Delivery time during the day / *afleweringstyd gedurende die dag*	Number of deliveries / *aantal aflwerings*	Cumulative frequency / *kumulatiewe frekwensie*	
$08:00 \leq x < 09:00$	5	5	
$09:00 \leq x < 10:00$	8	13	
$10:00 \leq x < 11:00$	12	25	
$11:00 \leq x < 12:00$	18	43	✓65
$12:00 \leq x < 13:00$	22	65	
$13:00 \leq x < 14:00$	25	90	
$14:00 \leq x < 15:00$	20	110	
$15:00 \leq x < 16:00$	15	125	
$16:00 \leq x < 17:00$	10	135	✓140
$17:00 \leq x < 18:00$	5	140	

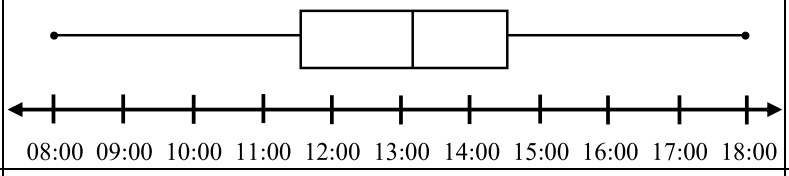
(2)

1.2

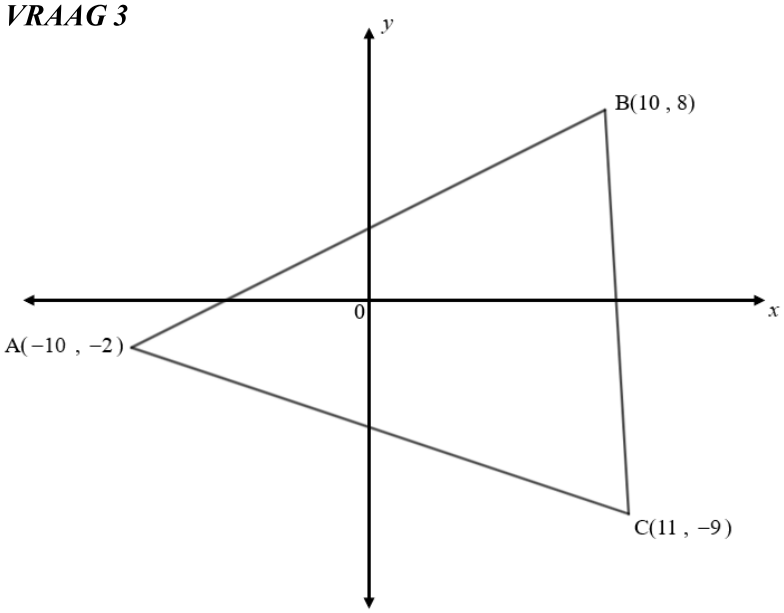


✓grounding/ *anker*
✓shape/ *vorm*
✓use of cumulative frequency/ *gebruik van kumulatiewe frekwensie*

(3)

1.3	5 Number summary / *5 getal opsomming* = {08:00 ; 11:36 ; 13:12 ; 14:36 ; 18:00}	✓min and/*en* max ✓Q_1 and/*en* Q_3 ✓Q_2	(3)
1.4		✓min and/*en* max ✓Q_1 and/*en* Q_3 ✓Q_2	(3)
1.5	Data is skewed to the left / *data is skeef na links*	✓answer/ *antwoord*	(1)
1.6	Standard deviation will stay the same / *standaardafwyking bly dieselfde*	✓answer/ *antwoord*	(1)
			[13]
QUESTION 2 / *VRAAG 2*			
2.1	Max $-$ 49 = 36 Max = 85	✓answer/ *antwoord*	(1)
2.2	IQR $= Q_3 - Q_1$ $Q_1 = 75 - 20 = 55$	✓answer/ *antwoord*	(1)
2.3	Two players: 8 + 64 = 72	✓answer/ *antwoord*	(1)
2.4	Skew to the right, $\bar{x} > Q_2$	✓answer/ *antwoord* ✓reason/ *rede*	(2)
2.5	$\bar{x} - Q_1 = 65 - 10,91 = 54,09$ 3 players	✓54,09 ✓answer/ *antwoord*	(2)
			[7]

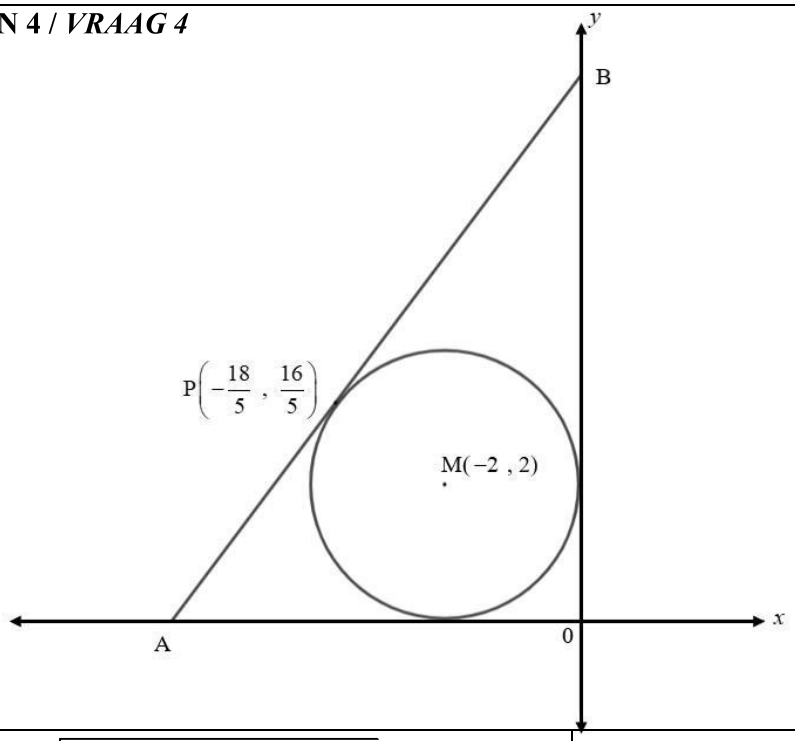
QUESTION 3 / *VRAAG 3*



3.1.1	$y-y_1=m(x-x_1)$ $y-(-9)=-2(x-11)$ $y+9=-2x+22$ $y=-2x+13$	$m(\text{AB})=\frac{8-(-2)}{10-(-10)}$ $m(\text{AB})=\frac{1}{2}$ $m(\text{altitude})=-2$	✓gradient AB ✓gradient altitude/ *hoogtelyn* ✓subst/ *vervang* in formula ✓answer/ *antwoord*	(4)
3.1.2	$y-y_1=m(x-x_1)$ $y-8=\frac{1}{2}(x-10)$ $y=\frac{1}{2}x+3$ …….. ① $y=-2x+13$ …….. ② Subst ① into ②: $\frac{1}{2}x+3=-2x+13$ $x+6=-4x+26$ $5x=20$ $x=4$ $y=5$ $\text{D}(4\,;5)$	$m(\text{AB})=\frac{1}{2}$	✓subst/ *vervang* in formula ✓equation/ *vgl* of AB ✓equating/ *gelykstel* ✓value of/*waarde v* x ✓value of/ *waarde v* y	(5)

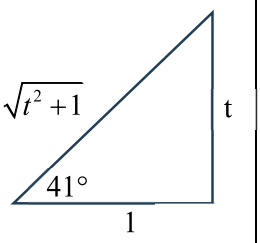
3.2	$AB = \sqrt{\left(10-\left(-10\right)\right)^2+\left(8-\left(-2\right)\right)^2}$ $AB = 10\sqrt{5}$ $CD = \sqrt{\left(11-4\right)^2+\left(-9-5\right)^2}$ $CD = 7\sqrt{5}$ $\text{area } \Delta ABC = \frac{1}{2}\text{base} \times \perp \text{height}$ $\text{area } \Delta ABC = \frac{1}{2}10\sqrt{5} \times 7\sqrt{5}$ $\text{area } \Delta ABC = 175$	✓subst/*vervang* in formula ✓length of/ *lengte v* AB ✓ length of/ *lengte v* CD ✓subst/ *vervang* in formula ✓answer/ *antwoord*	(5)
3.3	$AC = \sqrt{\left(-10-11\right)^2+\left(-2-\left(-9\right)\right)^2} = 7\sqrt{10}$ $\text{area } \Delta ABC = \frac{1}{2}AB \times AC \times \sin A$ $175 = \frac{1}{2}\left(10\sqrt{5}\right)\left(7\sqrt{10}\right)\sin A$ $\sin A = 0,707$ $\hat{A} = 45°$ **OR / *OF*** $m\left(AC\right) = \frac{-9-\left(-2\right)}{11-\left(-10\right)} = -\frac{1}{3}$ $AC:\ \tan\theta = -\frac{1}{3}$ $\theta = 180° - 18,43° = 161,57°$ $m\left(AB\right) = \frac{1}{2}$ $AB:\ \tan\alpha = \frac{1}{2}$ $\alpha = 26,57°$ $B\hat{A}C = 180° - \left(161,57° - 26,57°\right) = 45°$	✓subst in distance formula/ *vervang in afstand formule* ✓ length/ *lengte v* AC ✓ subst in area rule/ *vervang in opv reel* ✓ 0,707 ✓answer/ *antwoord* ✓ gradient AC ✓ 161,57° ✓ 26,57° ✓answer/ *antwoord*	(5)
			[19]

QUESTION 4 / *VRAAG 4*

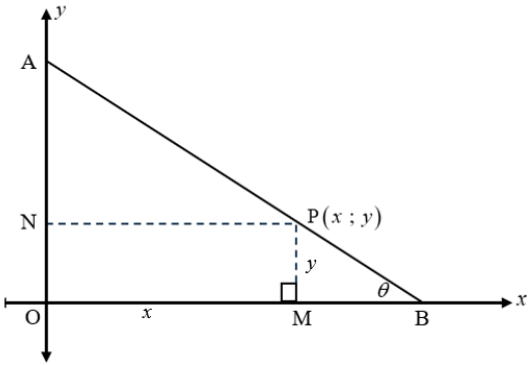


4.1	$MP = \sqrt{\left(-\frac{18}{5}-(-2)\right)^2+\left(\frac{16}{5}-2\right)^2}$ $= 2$ $(x-(-2))^2+(y-2)^2 = 2^2$ $(x+2)^2+(y-2)^2 = 4$	✓length of radius MP/ *lengte van radius MP* ✓substitution in formula/ *vervang in formule* ✓equation of circle/ *vergelyking van sirkel*	(3)
4.2	$m(MP) = \frac{2-\frac{16}{5}}{-2-\left(\frac{-18}{5}\right)} = -\frac{3}{4}$ $m(AB) = \frac{4}{3}$ $y-\left(\frac{16}{5}\right) = \frac{4}{3}\left(x-\left(-\frac{18}{5}\right)\right)$ $y-\frac{16}{5} = \frac{4}{3}x+\frac{24}{5}$ $y = \frac{4}{3}x+8$	✓ subst in formula/ *verv in formule* ✓ $m(MP)$ ✓ $m(AB)$ ✓ subst in line formula/ *vervang in lyn formule* ✓ answer/ *antwoord*	(5)
4.3	$(x+1)^2+(y-5)^2 = 1$	✓LHS/ *LK* ✓ RHS/ *RK*	(2)

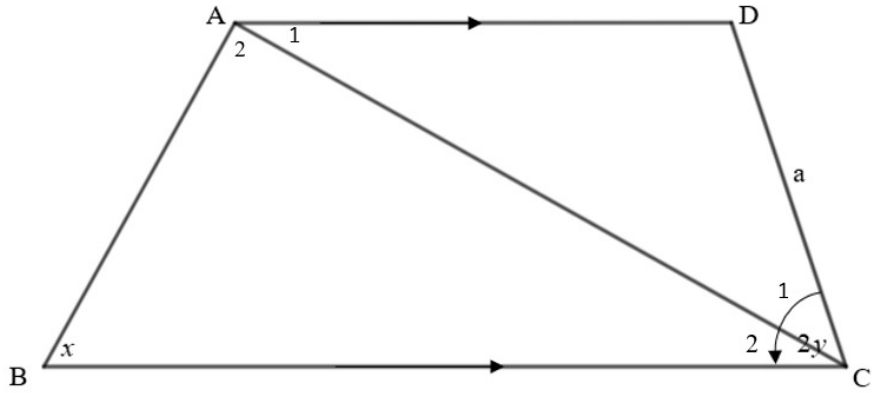
4.4	$(x+1)^2+(y-5)^2=1$ …..① $y=\frac{4}{3}x+8$ …..② $(x+1)^2+\left(\frac{4}{3}x+8-5\right)^2=1$ $x^2+2x+1+\left(\frac{4}{3}x+3\right)^2=1$ $x^2+2x+1+\frac{16}{9}x^2+8x+9-1=0$ $\frac{25}{9}x^2+10x+9=0$ $25x^2+90x+81=0$ $(5x+9)(5x+9)=0$ $x=-\frac{9}{5}$ $y=\frac{28}{5}$ $R\left(-\frac{9}{5};\frac{28}{5}\right)$	✓equating/ *gelykstel* ✓simplify/ *vereenvoudig* ✓std form/ *vorm* ✓factors/ *faktore* ✓ value of/*waarde v x* ✓value of/ *waarde v y*	(6)
4.5	B(0 ; 8) $BR=\sqrt{\left(0-\left(-\frac{9}{5}\right)\right)^2+\left(8-\frac{28}{5}\right)^2}$ $BR=3$ $BP=\sqrt{\left(0-\left(-\frac{18}{5}\right)\right)^2+\left(8-\frac{16}{5}\right)^2}=6$ $\frac{BR}{BP}=\frac{3}{6}=\frac{1}{2}$	✓B(0 ; 8) ✓subst in distance formula/ *vervang in afstand formule* ✓BR = 3 ✓BP = 6 ✓ratio	(5)
4.6	In ΔBPM : $\frac{BR}{BP}=\frac{1}{2}$ $\frac{NR}{MP}=\frac{1}{2}$ radii given/ *gegee* $\therefore \frac{BN}{BM}=\frac{1}{2}$ $\vert\vert\vert$ Δ	✓ratio ✓reasoning/ *rede*	(2)
			[23]

QUESTION 5 / *VRAAG 5*			
5.1	$\tan 319° = -\tan 41°$ $= -t$	✓def ✓answer/ *antwoord*	(2)
5.2	$\tan 41° = t$ $\sin 82°$ $= 2\sin 41°\cos 41°$ $= 2\left(\frac{t}{\sqrt{t^2+1}}\right)\left(\frac{1}{\sqrt{t^2+1}}\right)$ $= \frac{2t}{t^2+1}$  	✓ $\sqrt{t^2+1}$ ✓ double angle/ *dubbel hoek* ✓substitution/ *vervanging* ✓answer/ *antwoord*	(4)
5.3	$\cos 19° = \cos(60° - 41°)$ $= \cos 60°\cos 41° + \sin 60°\sin 41°$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{t^2+1}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{t}{\sqrt{t^2+1}}\right)$ $= \frac{1+\sqrt{3}t}{2\sqrt{t^2+1}}$ OR/*OF* $\cos 19° = \cos(49° - 30°)$ $= \cos 49°\cos 30° + \sin 49°\sin 30°$ $= \left(\frac{t}{\sqrt{t^2+1}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{t^2+1}}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{3}t+1}{2\sqrt{t^2+1}}$	✓ $60° - 41°$ ✓ compound angle/ *saamgestelde hoek* ✓subst 1st term/ *vervanging 1e term* ✓ subst 2nd term/ *vervanging 2e term* ✓ $49° - 30°$ ✓ compound angle/ *saamgestelde hoek* ✓subst 1st term/ *vervanging 1e term* ✓ subst 2nd term/ *vervanging 2e term*	(4)
			[10]

QUESTION 6 / *VRAAG 6*			
6.1	$\frac{4}{3}\cos^2 330° - \frac{1}{2\cos^2(-45°)} - \frac{1}{3}\sin(-30°)$ $= \frac{4}{3}(\cos 30°)^2 - \frac{1}{2(\cos 45°)^2} - \frac{1}{3}(-\sin 30°)$ $= \frac{4}{3}\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{1}{3}\left(-\frac{1}{2}\right)$ $= \frac{4}{3}\left(\frac{3}{4}\right) - \frac{1}{2\left(\frac{1}{2}\right)} + \frac{1}{6}$ $= 1 - 1 + \frac{1}{6} = \frac{1}{6}$	✓ $\cos 30°$ ✓ $\cos 45°$ ✓ $-\sin 30°$ ✓special angle values/ *spesiale hoekwardes* ✓simplification/ *vereenvoudig* ✓answer/ *antwoord*	(6)
6.2	$\frac{\sin^2\theta}{1-\cos\theta} - 1 = \cos\theta$ $\text{LHS} = \frac{\sin^2\theta}{1-\cos\theta} - 1$ $= \frac{1-\cos^2\theta}{1-\cos\theta} - 1$ $= \frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta} - 1$ $= 1 + \cos\theta - 1$ $= \cos\theta = \text{RHS}$ **OF/*OR*** $\frac{\sin^2\theta}{1-\cos\theta} - 1 = \cos\theta$ $\text{LHS} = \frac{\sin^2\theta}{1-\cos\theta} - 1$ $= \frac{\sin^2\theta - (1-\cos\theta)}{1-\cos\theta}$ $= \frac{\sin^2\theta - 1 + \cos\theta}{1-\cos\theta}$ $= \frac{-\cos^2\theta + \cos\theta}{1-\cos\theta}$ $= \frac{\cos\theta(-\cos\theta + 1)}{1-\cos\theta}$ $= \cos\theta = \text{RHS}$	✓square identity/ *vierkant identiteit* ✓factors/ *faktore* ✓simplification/ *vereenvoudig* ✓simplification/ *vereenvoudig* ✓square identity/ *vierkant identiteit* ✓factors/ *faktore*	(3)

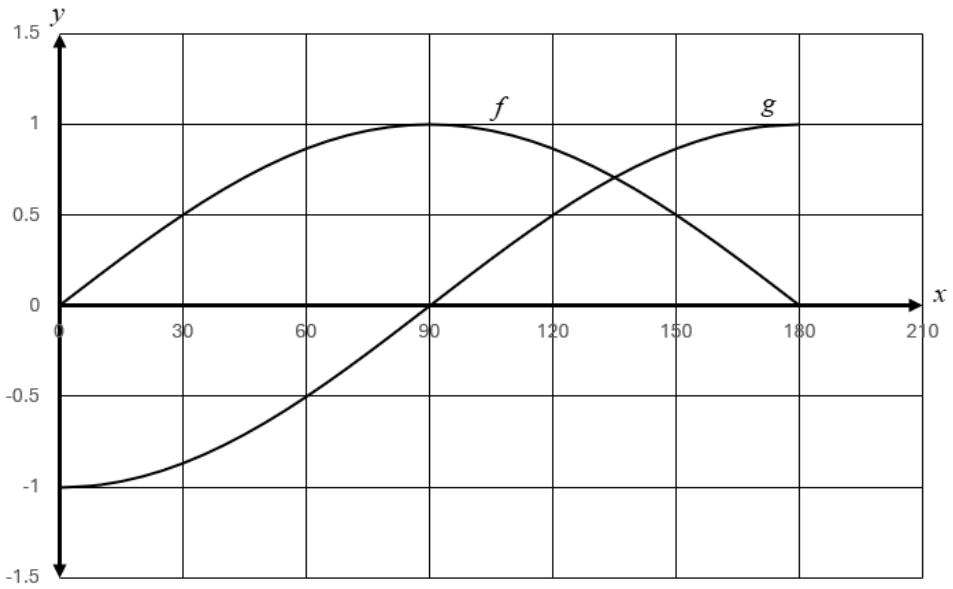
6.3	$2\sin^2 a+\sin a-\cos a=\sin 2a$ $2\sin^2 a+\sin a-\cos a=2\sin a\cos a$ $2\sin^2 a-2\sin a\cos a+\sin a-\cos a=0$ $2\sin a(\sin a-\cos a)+(\sin a-\cos a)=0$ $(\sin a-\cos a)(2\sin a+1)=0$ $\sin a=\cos a$ or/*of* $2\sin a=-1$ $\tan a=1$ or/*of* $\sin a=\frac{-1}{2}$ $a=45°+k.180°, k\in\mathbb{Z}$ NA	✓double angle/ *dubbelhoek* ✓simplification/ *vereenvoudig* ✓factors/ *faktore* ✓ $\tan a=1$ ✓ $\sin a=\frac{-1}{2}$ & NA (both) ✓ $45°+k.180°$ ✓ $k\in\mathbb{Z}$	(7)
6.4.1	 In ΔPMB: $\sin B=\frac{PM}{PB}$ $\sin\theta=\frac{y}{PB}$ $PB=\frac{y}{\sin\theta}$	✓substitution in trig ratio / *vervang in trig def* ✓answer / *antwoord*	(2)
6.4.2	$A\hat{P}N=\hat{B}=\theta$ corresponding $\angle$'s NP||OB In ΔANP: $\cos P=\frac{NP}{AP}$ $\cos\theta=\frac{x}{AP}$ $AP=\frac{x}{\cos\theta}$ $AB=AP+PB=\frac{y}{\sin\theta}+\frac{x}{\cos\theta}$	✓stating / *stel* $A\hat{P}N=\theta$ ✓substitution in trig ratio / *vervang in trig def* ✓answer / *antwoord* ✓AB = AP + PB	(4)
			[22]

QUESTION 7 / *VRAAG 7*



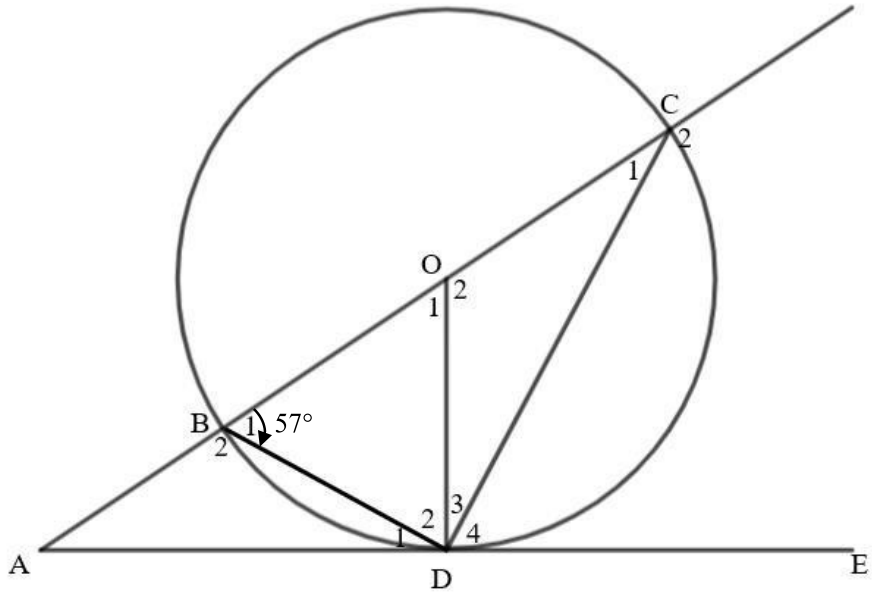
7.1	$D\hat{A}C = y$ $A\hat{D}C = 180° - 2y$	✓ y ✓ $180° - 2y$	(2)
7.2	$\frac{AC}{\sin D} = \frac{DC}{\sin D\hat{A}C}$ $\frac{AC}{\sin(180° - 2y)} = \frac{a}{\sin y}$ $\frac{AC}{\sin 2y} = \frac{a}{\sin y}$ $AC = \frac{a \sin 2y}{\sin y}$	✓subst in sine rule/ *vervang in sin reel* ✓ $\sin 2y$	(2)
7.3	$B\hat{A}C = 180° - (x + y)$ $\frac{AC}{\sin B} = \frac{BC}{\sin B\hat{A}C}$ $\frac{AC}{\sin B} = \frac{BC}{\sin(180° - (x+y))}$ $AC = \frac{BC \sin x}{\sin(x+y)}$	✓ $180° - (x + y)$ ✓subst in sine rule/ *vervang in sin reel*	(2)
7.4	$AC = \frac{a \sin 2y}{\sin y}$ and/*en* $AC = \frac{BC \sin x}{\sin(x+y)}$ $\frac{BC \sin x}{\sin(x+y)} = \frac{a \sin 2y}{\sin y}$ $BC = \frac{a \sin 2y \sin(x+y)}{\sin x \sin y}$ $BC = \frac{a2 \sin y \cos y \sin(x+y)}{\sin x \sin y}$ $BC = \frac{2a \cos y \sin(x+y)}{\sin x}$	✓equating/ *gelykstel* ✓BC subject of equation/ *BC die onderwerp van die vgl* ✓sine double angle/ *sin dubbelhoek*	(3)
			[9]

QUESTION 8 / *VRAAG 8*



8.1.1	$f(x) = \sin x$	✓equation/ *vgl*	(1)
8.1.2	$g(x) = -\cos x$	✓ equation/ *vgl*	(1)
8.2	$y \in [-1\ ;\ 1]$	✓critical values/ *kritieke waardes* ✓interval	(2)
8.3	360°	✓360°	(1)
8.4	1	✓1	(1)
8.5	$g(x).f'(x) < 0$ if $x \in [0°\ ;\ 180°]; x \neq 90°$ OR/*OF* $x \in [0°\ ;\ 90°) \cup (90°\ ;\ 180°]$	✓critical values/ *kritieke waardes* ✓intervals ✓critical values/ *kritieke waardes* ✓intervals	(2)
			[8]

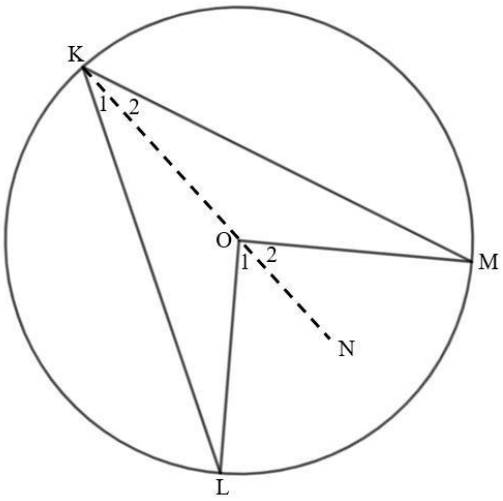
QUESTION 9 / *VRAAG 9*



9.1	$\hat{D}_4 + \hat{D}_3 = 90°$	rad $\perp$ tangent/ *raaklyn*	✓S ✓R	(2)
9.2	$\hat{D}_4 = 57°$ $\hat{D}_3 = 90° - 57° = 33°$	tan chord theorem/ *rklyn krd stelling*	✓S ✓R ✓answer/ *antw*	(3)
9.3	$\hat{D}_2 + \hat{D}_3 = 90°$ $\hat{D}_2 = 90° - 33° = 57°$ $\hat{C}_2 = \hat{B}_1 + \hat{D}_2 + \hat{D}_3$ $\hat{C}_2 = 57° + 90° = 147°$ **OR / *OF*** $\hat{B}_1 = \hat{D}_2 = 57°$ $\hat{C}_2 = \hat{B}_1 + \hat{D}_2 + \hat{D}_3$ $\hat{C}_2 = 57° + 90° = 147°$	$\angle$'s in semi-$\odot$ ext $\angle$ of Δ/ *buite* $\angle$ *van* Δ $\angle$'s opp equal sides/ $\angle$ *'e to gelyke sye* ext $\angle$ of Δ/ *buite* $\angle$ *van* Δ	✓S ✓R ✓S ✓S ✓S ✓R ✓S ✓answer/ *antw*	(4)
				[9]

QUESTION 10 / *VRAAG 10*

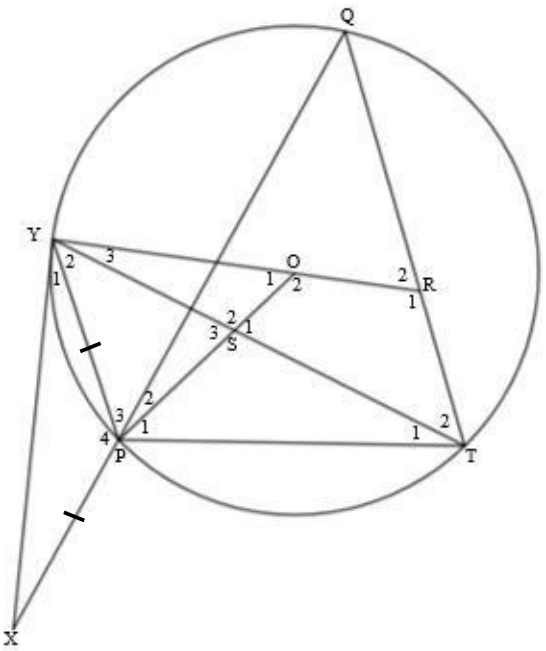
10.1



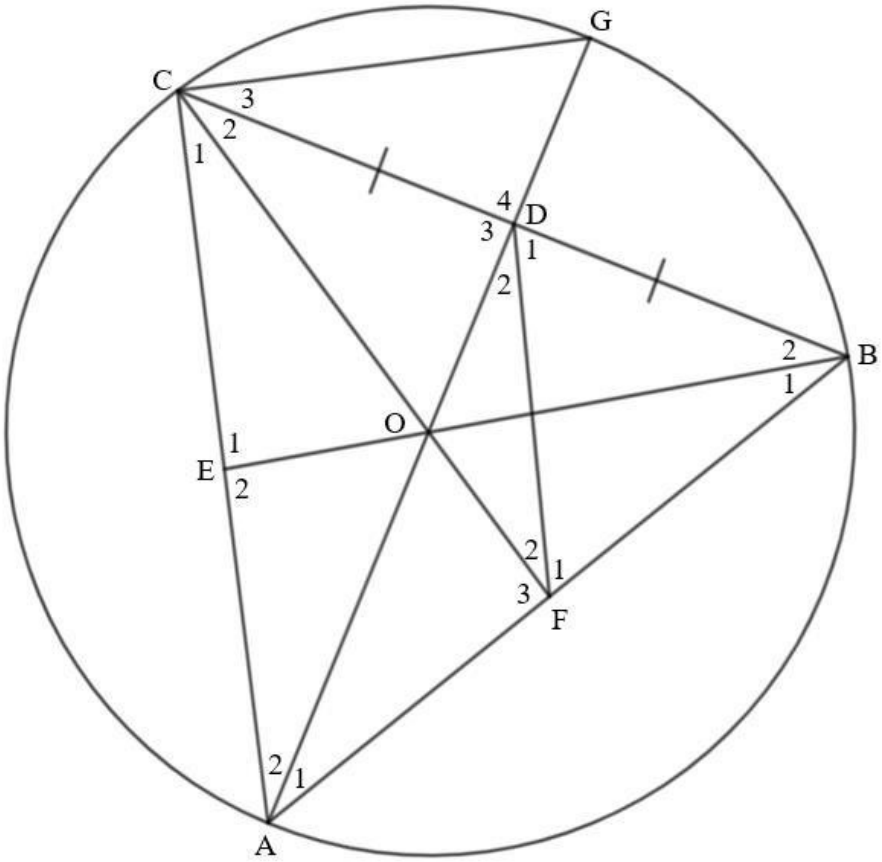
Draw line KO produced to N/ *Trek lyn KO en verleng na N*

$\hat{O}_1 = \hat{K}_1 + \hat{L}$		
$\hat{K}_1 = \hat{L}$	ext $\angle$ of Δ / *buite* $\angle$ *van* Δ	✓S✓R
$\hat{O}_1 = \hat{K}_1 + \hat{K}_1$	$\angle$'s opp equal sides/ $\angle$ *'e to gelyke sye*	✓S/R
$\hat{O}_1 = 2\hat{K}_1$		✓S
Similarly: $\hat{O}_2 = 2\hat{K}_2$		✓S
$\therefore L\hat{O}M = 2L\hat{K}M$		

(5)

10.2				
10.2.1	$\hat{X} = \hat{Y}_1 = m$ $\hat{T}_1 = \hat{Y}_1 = m$	$\angle$'s opp equal sides/ $\angle$ *'e to gelyke sye* tan-chord theorem/ *rklyn krd stelling*	✓S ✓R ✓S/R	(3)
10.2.2	$\hat{O}_1 = 2\hat{T}_1 = 2m$ $\hat{P}_3 = \hat{X} + \hat{Y}_1 = m + m = 2m$ $\hat{P}_3 = \hat{T}_2 = 2m$	$\angle$ at centre $= 2 \times \angle$ at circumf/ *midpt* $\angle = 2 \times$ *omtreks* $\angle$ ext $\angle$ of Δ / *buite* $\angle$ *van* Δ $\angle$'s in same segment/ $\angle$ *'e in dies segment*	✓S✓R ✓S✓R ✓S✓R	(6)
				[14]

QUESTION 11 / *VRAAG 11*



11.1	$\hat{D}_3 = 90°$ $\hat{F}_3 = 90°$ $\therefore \hat{D}_3 = \hat{F}_3$ AFDC is a cyclic quad/ *is kvh*	line from centre of ⊙ to midpt chord/ *lyn van midpt van ⊙ na midpt krd* given/ *gegee* converse ∠'s in same segment/ *omg ∠e in dies segment*	✓S ✓R ✓S ✓R	(4)
11.2	$\hat{C}_2 = \hat{A}_1$ $\hat{A}_1 = \hat{C}_3$ $\therefore \hat{C}_2 = \hat{C}_3$	∠'s in same segment/ *∠e in dies segment* ∠'s in same segment/ *∠e in dies segment*	✓S ✓R ✓S/R	(3)
11.3	In ΔABD and/*en* ΔCGD : (a) $\hat{A}_1 = \hat{C}_3$ (b) $\hat{B} = \hat{G}$ (c) $\hat{D}_1 = \hat{D}_4$ $\therefore \Delta ABD \,\|\|\, \Delta CGD$	 ∠'s in same segment/ *∠e in dies segment* ∠'s in same segment/ *∠e in dies segment* Vert opp ∠'s SSS	 ✓S ✓S ✓S	

	OR / *OF*			
	In ΔABD and/*en* ΔCGD :	$\angle$'s in same segment/ $\angle$*e in dies segment*		
	(a) $\hat{A}_1 = \hat{C}_3$	$\angle$'s in same segment/ $\angle$*e in dies segment*	✓S	
	(b) $\hat{B} = \hat{G}$	SSS	✓S	
	$\therefore \Delta ABD \text{ III } \Delta CGD$		✓R	(3)
11.4	$\frac{AB}{CG} = \frac{BD}{GD}$	$\Delta ABD \text{ III } \Delta CGD$	✓S/R	
	In ΔOCD and ΔGCD :			
	(a) $\hat{C}_2 = \hat{C}_3$	Proven/ *bewys*	✓S	
	(b) $\hat{D}_3 = \hat{D}_4$	$\hat{D}_3 = 90°$ proven/ *bewys*	✓S	
	(c) CD = CD	Common/ *gemeenskaplik*	✓S	
	$\therefore \Delta OCD \equiv \Delta GCD$	AAS/ *HHS*	✓R	
	$\therefore GD = DO$		✓S	
	$\frac{AB}{CG} = \frac{BD}{DO}$			(6)
				[16]

TOTAL/*TOTAAL*: 150