KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2025

MARKS: **100**

TIME: **2 hours**

N.B. This question paper consists of 7 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Given: The sum to n terms of an arithmetic sequence is $S_n = 3n^2 - 5n$.

1.1.1 Calculate the sum of the first 21 terms of this sequence. (2)

1.1.2 Determine the 22^{nd} term of this sequence. (2)

1.1.3 How many terms of this sequence must be added to obtain a sum of 8162? (4)

1.2 Consider the sequence: 7 ; 7 ; 7 ; 12 ; 7 ; 17 ; 7 ; 22; …

1.2.1 Determine the value of the 78^{th} term of this sequence. (2)

1.2.2 Calculate the sum of the first 103 terms of this sequence. (4)

[14]

QUESTION 2

2.1 Given: $\sum_{k=2}^{13} (-3)^k$

2.1.1 Write down the values of the first three terms of the series. (2)

2.1.2 Write down the value of the constant ratio. (1)

2.1.3 Will $\sum_{k=2}^{\infty} (-3)^k$ converge? Explain your answer. (2)

2.1.4 Calculate $\sum_{k=2}^{13} (-3)^k x$. Give your answer in terms of x. (3)

2.2 A quadratic sequence with a general term T_n has the following properties:

- $T_{29} = 1166$
- $T_n - T_{n-1} = 3n - 4$

Determine the value of the first term of the quadratic sequence. (6)

[14]

QUESTION 3

3.1 Given: $f(x)=\dfrac{2x+3}{-x-3}$

3.1.1 Show that $f(x)$ can also be written as $f(x)=\dfrac{3}{x+3}-2$. (2)

3.1.2 Hence, determine the coordinates of the intercepts of f with the axes. (3)

3.1.3 Sketch the graph of f. Clearly indicate the intercepts with both axes, as well as the asymptotes. (3)

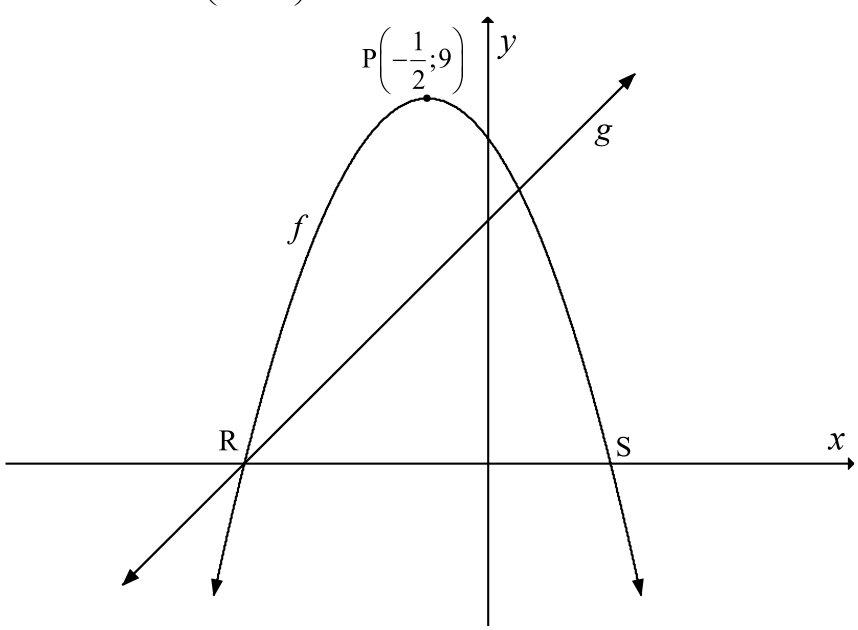
3.1.4 Determine the equation of the axis of symmetry of f, which is a decreasing function. (2)

3.1.5 Determine the value(s) of x for which:

(a) f is decreasing. (2)

(b) $f(x)\geq 0$. (2)

3.2 Sketched below is the parabola f and the straight line $g(x)=3x+6$. R and S are the x-intercepts of f, and $P\left(-\dfrac{1}{2};\,9\right)$ is its turning point. g has its x-intercept at R.



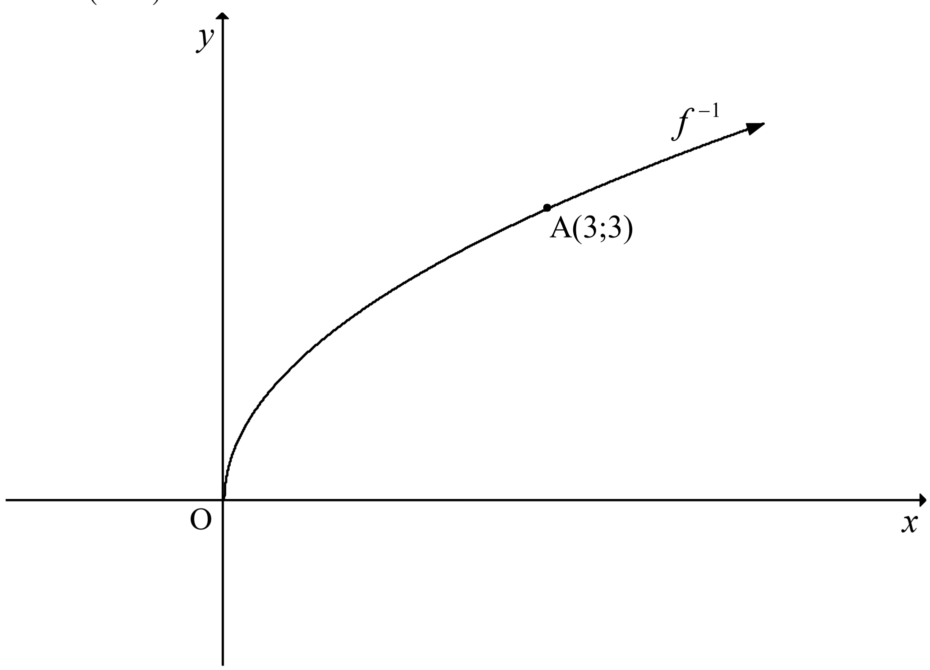
3.2.1 Calculate the coordinates of R. (2)

3.2.2 Determine the equation of the parabola. (4)

[20]

QUESTION 4

4.1 The graph of $f^{-1}(x)=\sqrt{3x}$, $x \geq 0$ is drawn in the sketch below. f^{-1} passes through the point $A(3\ ;3)$.



4.1.1 Determine the equation of f in the form $y=$ .... (3)

4.1.2 For which values of x will $f(x) \leq f^{-1}(x)$? (2)

4.2 Consider $g(x)=\left(\frac{1}{3}\right)^{x}$.

4.2.1 Determine the equation of g^{-1} in the form $y=$ .... (2)

4.2.2 Draw a sketch graph of g^{-1}, indicating any intercepts with the axes as well as one more point on the graph. (3)

4.2.3 The graph of $h(x)=a\left(\frac{1}{3}\right)^{x}+7$ passes through the point $(-2\ ;10)$. Calculate the value of a. (2)

4.2.4 Describe the translation from h to g. (3)

[15]

QUESTION 5

5.1 If $5\sin\beta - 4 = 0$ and $\beta \in (90° \, ; 270°)$, determine without the use of a calculator and with the aid of a diagram the values of:

5.1.1 $\cos\beta$ (3)

5.1.2 $\cos 2\beta$ (3)

5.1.3 $\sin 3\beta$ (4)

5.2 Simplify:

$$\frac{\sin(-180° - \theta).\tan(180° - \theta).\cos(-\theta)}{\cos^2(90° + \theta) + 3\sin^2\theta}$$ (6)

5.3 Prove the identity:

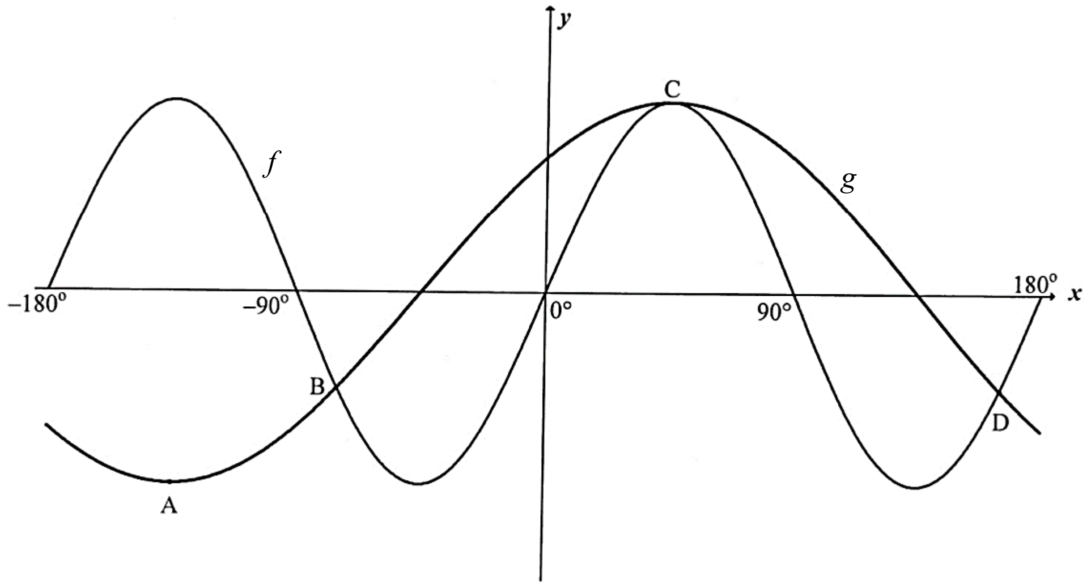
$$\frac{1}{8}(1 - \cos 4x) = \sin^2 x.\cos^2 x$$ (5)

[21]

QUESTION 6

6.1 Determine the general solution of $\cos(x-45°)=\sin 2x$. (4)

6.2 In the diagram, the graphs of $f(x)=\sin 2x$ and $g(x)=\cos(x-45°)$ are drawn for the interval $x\in[-180°\ ;\ 180°]$. A is a minimum point on graph g, and C is a maximum point on both graphs. The two graphs intersect at B, C and D.



6.2.1 Write down the period of g. (1)

6.2.2 Write down the coordinates of

(a) A (2)

(b) B (2)

6.2.3 Use the graphs to determine the values of x in the interval $x\in[0°\ ;\ 180°]$ for which $\dfrac{f(x)}{g(x)}<0$. (2)

6.2.4 Solve for x in the interval $x\in[0°\ ;\ 180°]$ if $\sin 2x\geq\dfrac{1}{\sqrt{2}}(\cos x+\sin x)$. Show all your working. (5)

[16]

TOTAL: 100 marks

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$