DISTRICT PAPER

GRADE 12

END OF TERM 1 TEST 2025

NATIONAL SENIOR CERTIFICATE

JOHANNESBURG NORTH [D10]

MATHEMATICS

MARKS: 100

TIME: 2 hours

EXAMINER: MR S. NDLOVU
MODERATOR: MS B. RANTAO

This question paper consists of 8 printed pages including the cover page.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions. **Answer ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non- graphical), unless stated otherwise.
5. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
6. Diagrams are **NOT** necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x, in each of the following:

1.1.1 $(x-3)(x+11)=0$ (2)

1.1.2 $3x^2-1=-7x$ (correct to TWO decimal places) (5)

1.1.3 $2x^2+x-6\geq 0$ (4)

1.2 Solve simultaneously for x and y in the following equations:

$x+2y=5$ and $x^2-3xy+y^2=-1$ (6)

[17]

QUESTION 2

2.1 Prove that for any Arithmetic Sequence the sum of the first n terms is given by

$S_n=\frac{n}{2}[2a+(n-1)d]$, if the first term is a and the common difference is d. (4)

2.2 Given the arithmetic series: $14+21+28+35+\cdots+287$.

2.2.1 How many terms are there in the series? (2)

2.2.2 Calculate the sum of all natural numbers from 12 to 115 that are NOT divisible by 7. (4)

2.3 Show that:

$$\sum_{k=2}^{n}(6k-2)=3n^2+n-4$$ (3)

[13]

QUESTION 3

Given the geometric series: $8(p-3)+4(p^2-9)+2(p^3-3p^2-9p-27)+\cdots$

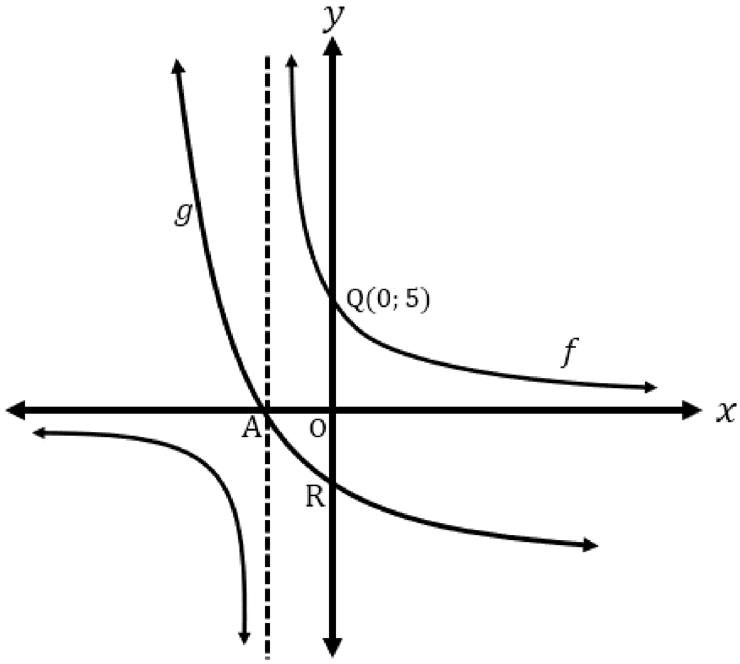
3.1 Determine the values of p for which the series will converge. (3)

3.2 If $p=-2$, determine the sum to infinity of the series. (4)

[7]

QUESTION 4

The diagram below shows the graphs of $f(x) = \frac{a}{x+p} + q$ and $g(x) = \left(\frac{1}{3}\right)^x - 9$.
Graph f cuts the y −axis at Q(0 ; 5). Graph g cuts the asymptotes of graph f at point A.



4.1 Write down the asymptote of graph g. (1)

4.2 Determine the coordinates of point R. (2)

4.3 Determine the values of a, p and q in graph f. (3)

4.4 If $h(x) = f(-2x) + 9$, determine $h^{-1}(x)$ and write your answer in the $y = \cdots$ (4)

4.5 Sketch the graph of $h^{-1}(x)$ showing intercept(s) with the axes and at least one more point on the graph. (2)

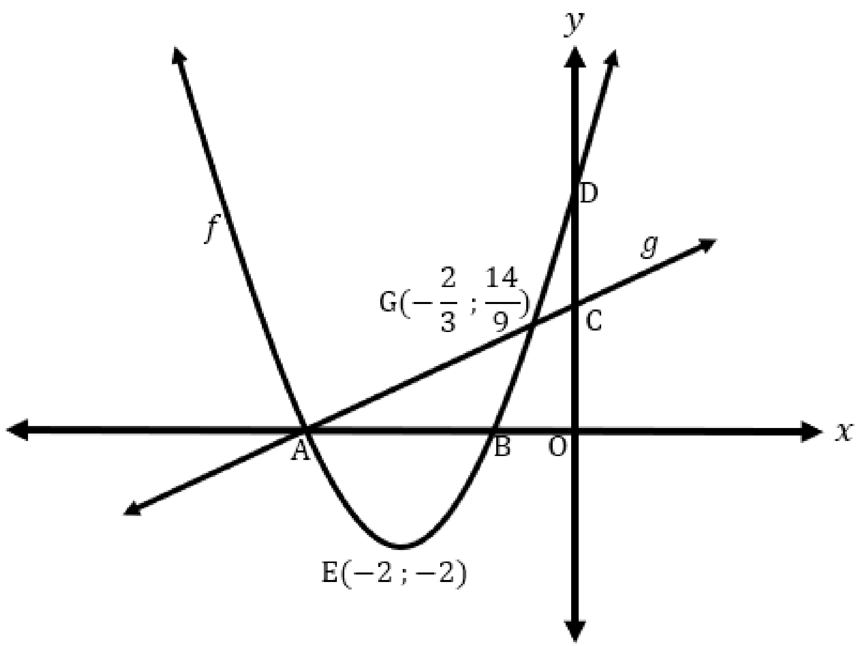
4.6 Determine the values of x for which $h^{-1}(x) \leq 2$. (2)

[14]

QUESTION 5

The functions $f(x) = a(x-p)^2 + q$ and $g(x) = \frac{2}{3}x + k$ are drawn below, with f passing through A, E, B and D. A and B are the x-intercepts of f.

E$(-2\,;-2)$ is the turning point of graph f. Graph g passes through point A, G$\left(\frac{-2}{3}\,;\frac{14}{9}\right)$ and cuts the $y-$axis at C. A and G are points of intersection of graphs f and g.



5.1 Determine the value of k in graph g. (2)

5.2 Determine the equation of graph f by finding the values of a, p and q. (4)

5.3 Determine the equation of $h(x)$, a line perpendicular to graph g and passing through point D. (3)

5.4 Determine the values of x for which $f(x) \geq g(x)$ (3)

5.5 It is further given that graph $m(x) = 3^{-f(x)}$, determine the maximum value of the graph m. (2)

5.6 Determine the average gradient of graph f between $x = -5$ and $x = -1$ (4)

[18]

QUESTION 6

Given: $\sin 21° = k$

Without using a calculator, determine in terms of k, each of the following.

6.1 $\cos 21°$ (3)

6.2 $\tan 249°$ (3)

6.3 $\cos 138°$ (4)

[10]

QUESTION 7

7.1 Given that $P = \cos(45° + x) + \sin(45° - x)$

7.1.1 Simplify expression P as far as possible. (3)

7.1.2 Hence, determine **without the use of a calculator**, the value of P, if $x = 60°$ (2)

7.2 Prove that:

$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ (5)

7.3 Determine the general solution of the equation:

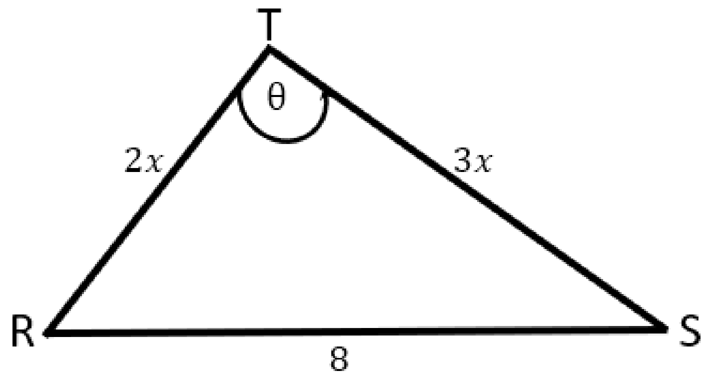
$4 - 5\cos x = 2\sin^2 x$ (5)

[15]

QUESTION 8

In ΔTSR ,sketched below, TS = $3x$ units, TR = $2x$ units and SR= 8 units.

$S\hat{T}R = \theta$.



8.1 Show that

$\cos\theta = \frac{13x^2-64}{12x^2}$ (3)

8.2 If it is given that $x = 2.5$, determine the area of ΔTSR. (3)

[6]

INFORMATION SHEET: MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = P(1 + in)$

$A = P(1 - in)$

$A = P(1 + i)^n$

$A = P(1 - i)^n$

$\sum_{i=1}^{n} 1 = n$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$T_n = a + (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$; $r \neq 1$

$S_\infty = \frac{a}{1-r}$; $r \neq 1$

$F_v = \frac{x[(1+i)^n - 1]}{i}$

$P_V = \frac{x[1 - (1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

Area of $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$

$\sigma^2 = \frac{\sum_i^n (xi - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n((S)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$

$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$