basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2025

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 26 pages./
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • MEETKUNDE	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

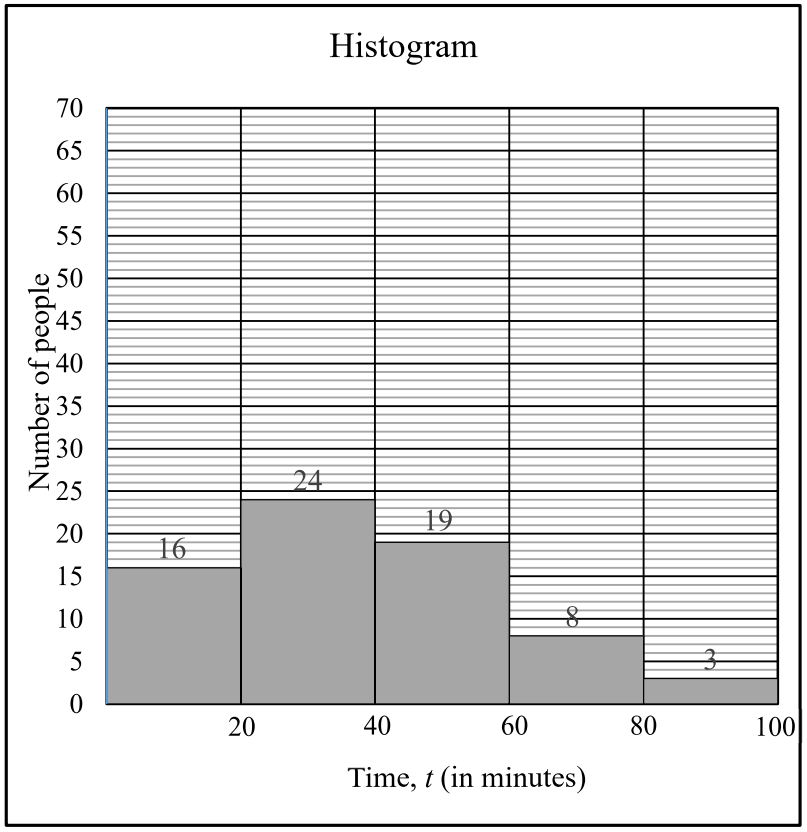
QUESTION/*VRAAG* 1

AGE OF CAR (IN YEARS)	SELLING PRICE OF CAR (IN RANDS)
2	293 000
3	265 000
3	256 000
4	219 000
4	241 000
4	246 000
6	226 000
6	176 000
7	154 000
7	180 000
8	148 000

1.1	$a = 331\ 397{,}20$ $b = -22\ 988{,}32$ $\hat{y} = 331\ 397{,}20 - 22\ 988{,}32x$	✓ $a = 331\ 397{,}20$ ✓ $b = -22\ 988{,}32$ ✓ equation (3)
1.2	$\hat{y} = 331\ 397{,}20 - 22\ 988{,}32(5)$ $= 216\ 455{,}60$ **OR/*OF*** $\hat{y} = 216\ 455{,}61$ (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.3	The strong correlation ($r = -0{,}95$) suggests that the data points lie close to the regression line. Therefore, the prediction will be valid./ *'n Sterk korrelasie* ($r = -0{,}95$) *dui aan dat die punte naby aan die regressielyn lê.* *Dus, die voorspelling is geldig.*	✓ strong correlation **OR** $r = -0{,}95$ ✓ answer (2)
1.4	The average decrease per year is R22 988,32. *Die gemiddelde afname per jaar is* R 22 988,32.	✓ answer (1)
		[8]

QUESTION/*VRAAG* 2

TIME, t (IN MINUTES)	CUMULATIVE FREQUENCY
$0 < t \leq 20$	16
$0 < t \leq 40$	40
$0 < t \leq 60$	59
$0 < t \leq 80$	67
$0 < t \leq 100$	70

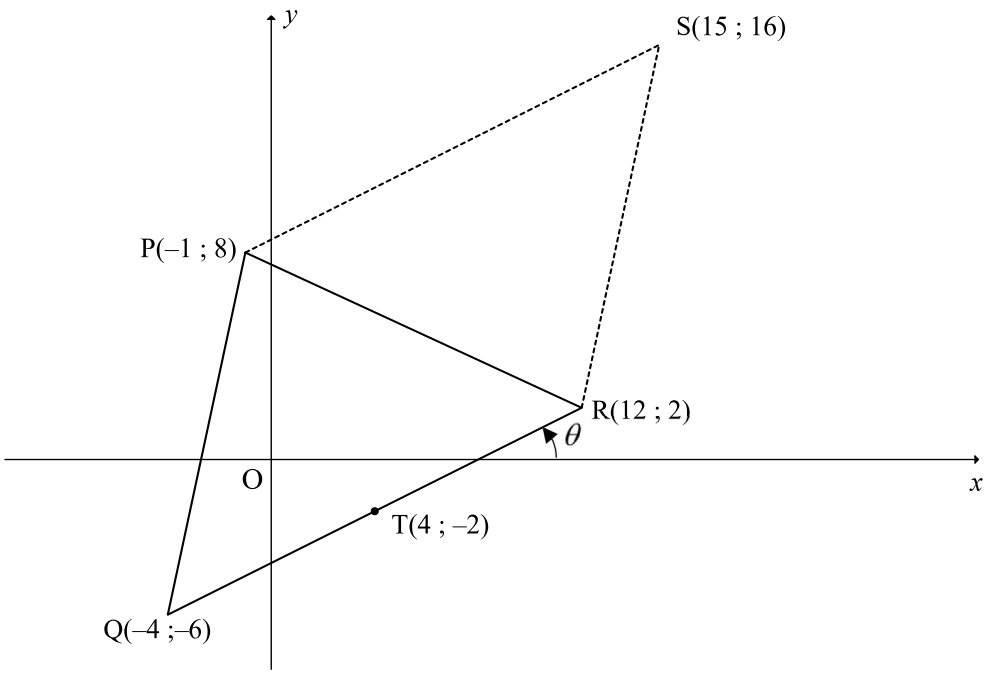
2.1.1	70	✓ 70 (1)
2.1.2	No. of people = 67 – 40 = 27	✓ 67 – 40 ✓ 27 (2)
2.1.3		✓ two frequencies correct ✓ all frequencies correct ✓ no gaps between bars (3)
2.1.4	Skewed to the right **OR** positively skewed *Skeef na regs* **OF** *positief skeef*	✓ answer (1)

2.2

11	14	19	20	8	10	2	14

	$\frac{11+14+19+20+8+10+2+14+x}{9} = 12$ $x + 98 = 108$ $x = 10$ The 9th player scored 10 points $\sigma = 5,23 \qquad (5,22812)$ $(\bar{x}-\sigma\ ;\ \bar{x}+\sigma) = (12-5,23;\ 12+5,23)$ $= (6,77;17,23)$ 3 players' points were outside one standard deviation of the mean. 3 *spelers se punte aangeteken lê buite een standaardafwyking van die gemiddeld.*	✓ equating using mean ✓ answer ✓ standard deviation ✓ interval ✓ answer (5)
		[12]

QUESTION/*VRAAG* 3

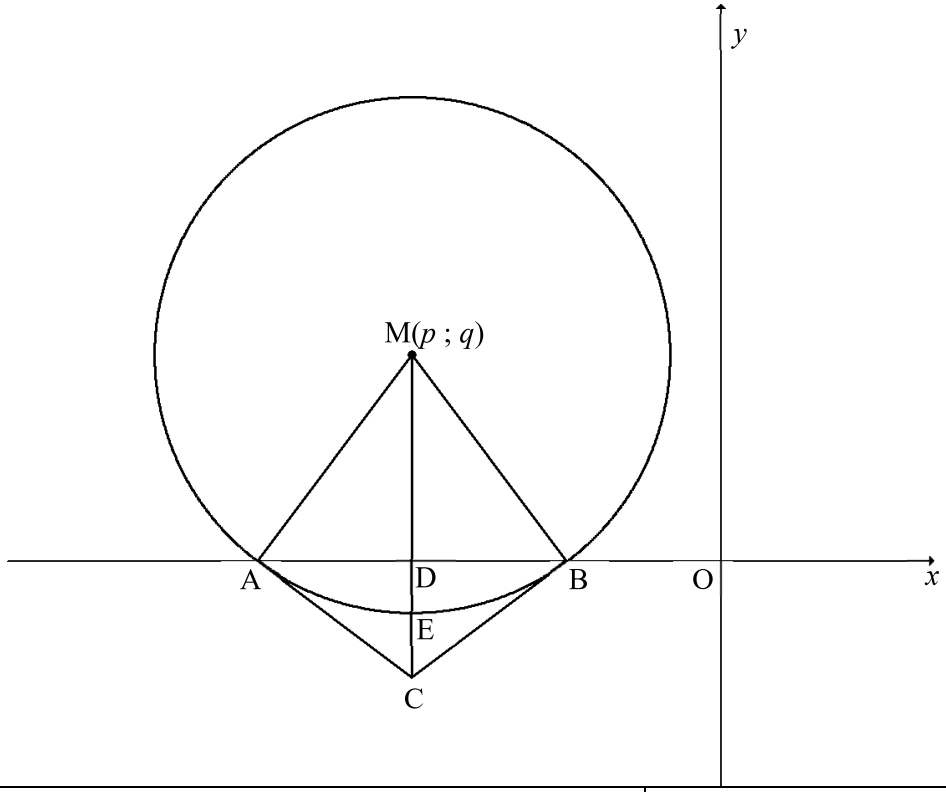


3.1	$QR=\sqrt{(-4-12)^2+(-6-2)^2}$ $=\sqrt{320}=8\sqrt{5}$ units	✓ $QR=\sqrt{(-4-12)^2+(-6-2)^2}$ ✓ answer (2)
3.2	$m_{QR}=\frac{-6-2}{-4-12}$ OR $m_{QR}=\frac{2-(-6)}{12-(-4)}$ $m_{QR}=\frac{1}{2}$ $m_{QR}=\frac{1}{2}$	✓ correct substitution of Q(–4 ; –6) & R(12 ; 2) into gradient formula ✓ answer (2)
3.3	$m_{QR}=\frac{1}{2}$ $\tan\theta=\frac{1}{2}$ $\theta=26,57°$	✓ $\tan\theta=m_{QR}$ ✓ answer (2)
3.4	$m_{QR}=\frac{1}{2}$ $-6=\frac{1}{2}(-4)+c$ **OR** $y-2=\frac{1}{2}(x-12)$ $c=-4$ $y-2=\frac{1}{2}x-6$ $y=\frac{1}{2}x-4$ $y=\frac{1}{2}x-4$	✓ correct substitution of gradient and point Q(–4 ; –6) or R(12 ; 2) ✓ answer (2)
3.5	$Q\rightarrow R:(x;y)\rightarrow(x+16;y+8)$ $\therefore S(15;16)$	✓ $x_S=15$ ✓ $y_S=16$ (2)

3.6	$m_{QR} = \frac{1}{2}$ $m_{PT} = -2$ Equation of PT: $y = -2x + c$ **OR** $y - y_1 = -2(x - x_1)$ $8 = -2(-1) + c$ **OR** $y - 8 = -2(x - (-1))$ $c = 6$ $y - 8 = -2x - 2$ $y = -2x + 6$ $y = -2x + 6$ $-2x + 6 = \frac{1}{2}x - 4$ $-4x + 12 = x - 8$ $5x = 20$ $x = 4$ $y = \frac{1}{2}(4) - 4$ $y = -2$ $T(4;-2)$ **OR** $PQ = \sqrt{(-4-(-1))^2 + (-6-8)^2} = \sqrt{205}$ $PR = \sqrt{(12-(-1))^2 + (2-8)^2} = \sqrt{205}$ $\therefore$ ΔPQR is isosceles / ΔPQR *is 'n gelykbenige* Δ $\therefore$ $\perp$ height bisects the base QR / $\perp$ *hoogte halveer die basis* QR $\therefore$ T is midpoint of QR / T *is middelpunt van* QR $\therefore T(4;-2)$	✓ m_{PT} ✓ equation of PT ✓ equation QR = equation PT ✓ simplification ✓ $T(x_T ; y_T)$ (5) ✓ $PQ = \sqrt{205}$ ✓ $PR = \sqrt{205}$ ✓ ΔPQR is isosceles ✓ $\perp$ height bisects the base QR ✓ $T(4;-2)$ (5)
3.7	$PT = \sqrt{(4-(-1))^2 + (-2-8)^2}$ $PT = \sqrt{125} = 5\sqrt{5}$ units $= 11{,}18$ units Area of PQRS = QR.PT $= (8\sqrt{5})(5\sqrt{5})$ $= 200$ units^2 **OR**	✓ length of PT ✓ substitution of QR and PT ✓ answer (3)

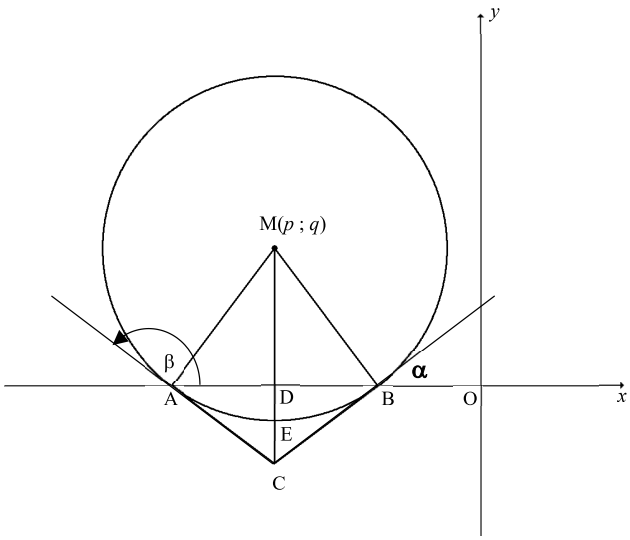
	$PT = \sqrt{\left(4-(-1)\right)^2+(-2-8)^2}$ $PT = \sqrt{125} = 5\sqrt{5}$ units $= 11{,}18$ units Area of $\Delta PQR = \frac{1}{2}\left(8\sqrt{5}\right)\left(5\sqrt{5}\right)$ $= 100$ units^2 Area of PQRS $= 2\times$ Area of ΔPQR $= 200$ units^2	✓ length of PT ✓ substitution of QR and PT ✓answer (3)
		[18]

QUESTION/*VRAAG* 4



4.1	$p = -6$	✓ $p = -6$ (1)
4.2	$M\hat{D}B = 90°$ [MC ⊥ y-axis] $AM = ME = q + 1$ [radii] $MD = q$ $AM^2 = AD^2 + MD^2$ [Pythagoras] $(q+1)^2 = (q-1)^2 + q^2$ $q^2 + 2q + 1 = q^2 - 2q + 1 + q^2$ $q^2 - 4q = 0$ $q(q-4) = 0$ $q \neq 0$ or $q = 4$	✓ $AM = q + 1$ ✓ $MD = q$ ✓ substitution into Pythagoras ✓ standard form (4)
4.3	AM = 5 units $(x+6)^2 + (y-4)^2 = 25$	✓ LHS ✓ RHS (2)
4.4	3 units	✓ answer (1)

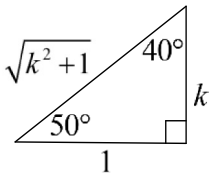
4.5	$(x+6)^2+(0-4)^2=25$ $(x+6)^2=9$ $x+6=3$ or $x+6=-3$ $x=-3$ $x=-9$ A$(-9\ ;\ 0)$ B$(-3\ ;\ 0)$ **OR** $(x+6)^2+(0-4)^2=25$ $x^2+12x+36+16-25=0$ $x^2+12x+27=0$ $(x+3)(x+9)=0$ $x=-3$ or $x=-9$ A$(-9\ ;\ 0)$ B$(-3\ ;\ 0)$ **OR** $q-1=3$ DB = AD = 3 [line from centre ⊥ to chord/ *lyn vanuit midpt ⊥ op koord*] A$(-9\ ;\ 0)$ B$(-3\ ;\ 0)$	✓ substituting $y=0$ into equation of circle ✓ coordinates of A ✓ coordinates of B (3) ✓ substituting $y=0$ into equation of circle ✓ coordinates of A ✓ coordinates of B (3) ✓ DB = 3 ✓ coordinates of A ✓ coordinates of B (3)
4.6	$m_{MB}=\frac{4-0}{-6-(-3)}$ $=-\frac{4}{3}$ $m_{BC}=\frac{3}{4}$ $y=\frac{3}{4}x+c$ $0=\frac{3}{4}(-3)+c$ $c=\frac{9}{4}$ $y=\frac{3}{4}x+\frac{9}{4}$ **OR** $y-y_1=\frac{3}{4}(x+x_1)$ $y-0=\frac{3}{4}[x-(-3)]$ $y=\frac{3}{4}(x+3)$ $y=\frac{3}{4}x+\frac{9}{4}$	✓ m_{MB} ✓ m_{BC} ✓ substitution of gradient BC and coordinates of B ✓ answer (4)
4.7	C$\left(-6\ ;-\frac{9}{4}\right)$	✓ x_C ✓ y_C (2)



4.8	$\tan\alpha = \frac{3}{4}$ $\alpha = 36,87°$	✓ $\alpha = 36,87°$
	$m_{AC} = -\frac{3}{4}$ $\tan\beta = -\frac{3}{4}$	✓ $\tan\beta = m_{AC}$
	$\beta = 180° - 36,87°$ $\beta = 143,13°$	✓ value of β
	$\therefore A\hat{C}B = 106,26°$	✓ answer (4)
	OR	
	$\tan M\hat{A}B = m_{MA} = \frac{4}{3}$	
	$M\hat{A}B = 53,13°$	✓ $M\hat{A}B$
	$A\hat{M}D = 90° - 53,13°$ $A\hat{M}D = 36,87°$	✓ $A\hat{M}D$
	$M\hat{A}C = 90°$ [tangent ⊥ radius / *raaklyn⊥radius*]	
	$A\hat{C}M = 53,13°$	✓ $A\hat{C}M$
	$\therefore A\hat{C}B = 106,26°$ [property of kite /*eienskappe van vlieër*]	✓ answer (4)
	OR	
	$\tan A\hat{C}D = \frac{AD}{DC}$	✓ trig ratio in ΔACD or ΔBCD
	$\tan A\hat{C}D = \frac{3}{\frac{9}{4}} = \frac{4}{3}$	✓ $\tan A\hat{C}D$
	$A\hat{C}D = 53,13°$	✓ $A\hat{C}D$
	$\therefore A\hat{C}B = 106,26°$ [property of kite/*eienskappe van vlieër*]	✓ answer (4)

	OR AB = 6 units $AC = BC = \frac{15}{4}$ units [tangent from same point/ *raaklyne vanuit dieselfde punt*] $(AB)^2 = (AC)^2 + (BC)^2 - 2(AC)(BC)\cos \hat{C}$ $(6)^2 = \left(\frac{15}{4}\right)^2 + \left(\frac{15}{4}\right)^2 - 2\left(\frac{15}{4}\right)\left(\frac{15}{4}\right)\cos \hat{C}$ $\cos A\hat{C}B = -0,28$ $A\hat{C}B = 106,26°$ **OR** $\tan M\hat{A}B = m_{MA} = \frac{4}{3}$ $M\hat{A}B = 53,13°$ AMBC is a cyclic quad/ AMBC *is 'n kvh* $\therefore M\hat{C}B = 53,13°$ [∠s in the same seg/*∠e in dies segm*] $\therefore A\hat{C}B = 106,26°$ [property of kite/*eienskappe v vlieër*]	✓ AC = BC ✓ substitution into cosine-rule ✓ simplification ✓ answer (4) ✓ MÂB ✓ AMBC is a cyclic quad/*kvh* ✓ MĈB ✓ answer (4)
		[21]

QUESTION/*VRAAG* 5

5.1.1	$r^2 = k^2 + 1^2$ [Pythagoras] $r = \sqrt{k^2+1}$  $\cos 40°$ $= \frac{k}{\sqrt{k^2+1}}$	✓ third side = $\sqrt{k^2+1}$ ✓ answer (2)
5.1.2	$\frac{2\sin 25°\cos 25°}{-2+4\sin^2 25°}$ $= \frac{\sin 50°}{-2(1-2\sin^2 25°)}$ $= \frac{\sin 50°}{-2\cos 50°}$ $= \left(\frac{k}{\sqrt{k^2+1}}\right) \div \left(\frac{-2}{\sqrt{k^2+1}}\right)$ **OR** $= -\frac{1}{2}\tan 50°$ $= -\frac{1}{2}k$	✓ $\sin 50°$ ✓ $-2(1-2\sin^2 25°)$ ✓ double angle ✓ subst **OR** quotient identity ✓ answer (5)
5.1.3	$\sin 10° = \sin(50° - 40°)$ $= \sin 50°\cos 40° - \cos 50°\sin 40°$ $= \left(\frac{k}{\sqrt{k^2+1}}\right)\left(\frac{k}{\sqrt{k^2+1}}\right) - \left(\frac{1}{\sqrt{k^2+1}}\right)\left(\frac{1}{\sqrt{k^2+1}}\right)$ $= \frac{k^2-1}{k^2+1}$ **OR** $\sin 10°$ $= \cos 80°$ $= \cos 2(40°)$ $= 2\cos^2 40° - 1$ $= 2\left(\frac{k}{\sqrt{k^2+1}}\right)^2 - 1$ $= \frac{2k^2}{k^2+1} - 1$ $= \frac{k^2-1}{k^2+1}$ **OR**	✓ $\sin 10° = \sin(50° - 40°)$ ✓ correct expansion ✓ first term ✓ second term (4) ✓ $\sin 10° = \cos 80°$ ✓ correct expansion ✓✓ substitution (4)

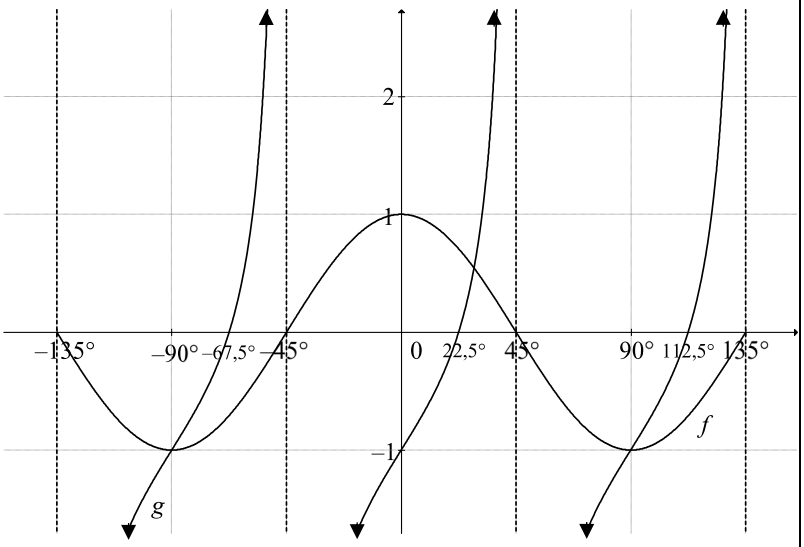
	$\sin 10° = \sin(60° - 50°)$ $= \sin 60° \cos 50° - \cos 60° \sin 50°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{k^2+1}}\right) - \left(\frac{1}{2}\right)\left(\frac{k}{\sqrt{k^2+1}}\right)$ $= \frac{\sqrt{3}-k}{2\sqrt{k^2+1}}$	✓ $\sin 10° = \sin(60° - 50°)$ ✓ correct expansion ✓ first term ✓ second term (4)
5.2.1	$\frac{\sin(540° + x).\cos(90° + x)}{\sin(-x)}$ $= \frac{(-\sin x)(-\sin x)}{(-\sin x)}$ $= -\sin x$	✓ $\sin(540° + x) = -\sin x$ ✓ $\cos(90° + x) = -\sin x$ ✓ $\sin(-x) = -\sin x$ ✓ answer (4)
5.2.2	$x \in (180°; 360°)$ **OR** $180° < x < 360°$	✓✓ $x \in (180°; 360°)$ (2) ✓✓ $180° < x < 360°$ (2)
		[17]

QUESTION/VRAAG 6

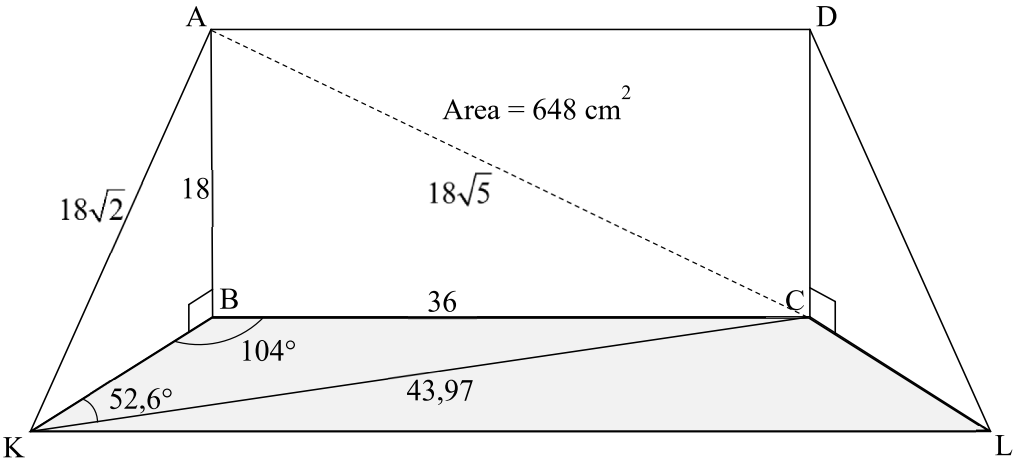
6.1	$\text{LHS} = \left[\tan(180° - x)\right](1 - \cos^2 x) + \cos^2 x$	
	$= (-\tan x)(\sin^2 x) + \cos^2 x$	✓ $\tan(180° - x) = -\tan x$
	$= \left(-\frac{\sin x}{\cos x}\right)(\sin^2 x) + \cos^2 x$	✓ $1 - \cos^2 x = \sin^2 x$ ✓ quotient identity
	$= -\frac{\sin^3 x}{\cos x} + \cos^2 x$	
	$= \frac{\sin^3 x - \cos^3 x}{-\cos x}$	✓ simplification to a single fraction
	$= \frac{(\sin x - \cos x)(\sin^2 x + \sin x.\cos x + \cos^2 x)}{-\cos x}$	✓ factors for a difference of cubes
	$= \frac{(\sin x - \cos x)(1 + \sin x.\cos x)}{-\cos x}$	✓ $\sin^2 x + \cos^2 x = 1$
	$= \text{RHS}$	(6)
	OR	
	$\text{RHS} = \frac{(\sin x - \cos x)(1 + \sin x.\cos x)}{-\cos x}$	
	$= \frac{(\sin x - \cos x)(\cos^2 x + \sin^2 x + \sin x.\cos x)}{-\cos x}$	✓ $1 = \sin^2 x + \cos^2 x$
	$= \frac{\sin x\cos^2 x + \sin^3 x + \sin^2 x\cos x - \cos^3 x - \sin^2 x\cos x - \sin x.\cos^2 x}{-\cos x}$	✓ expansion
	$= \frac{\sin^3 x - \cos^3 x}{-\cos x}$	✓ simplification
	$= \frac{\sin^3 x}{-\cos x} + \cos^2 x$	✓ split fraction
	$= \frac{-\sin x}{\cos x}(\sin^2 x) + \cos^2 x$	✓ quotient identity
	$= -\tan x(1 - \cos^2 x) + \cos^2 x$	✓ $\sin^2 x = 1 - \cos^2 x$
	$= \text{LHS}$	(6)

6.2	$\sin^2 x\ ;\ \cos^2 x\ ;\ \frac{1}{2}\sin 2x$	
	$\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$	✓ $\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$
	$\cos^2 x-\sin^2 x=\frac{1}{2}(2\sin x\cos x)-\cos^2 x$	✓ $\sin 2x=2\sin x\cos x$
	$\cos^2 x-\sin^2 x=\sin x\cos x-\cos^2 x$	
	$2\cos^2 x-\sin x\cos x-\sin^2 x=0$	✓ standard form
	$(2\cos x+\sin x)(\cos x-\sin x)=0$	✓ factors
	$2\cos x=-\sin x$ or $\cos x=\sin x$ $\tan x=-2$ or $\tan x=1$	✓ both equations
	ref $\angle = 63,43°$ or ref $\angle = 45°$	
	$x=116,57°+k.180°$ or $x\neq 45°+k.180°;k\in Z$	✓ $x=116,57°$ ✓ $116,57°+k.180°;k\in Z$
	OR	(7)
	$\sin^2 x\ ;\ \cos^2 x\ ;\ \frac{1}{2}\sin 2x$	
	$\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$	✓ $\cos^2 x-\sin^2 x=\frac{1}{2}\sin 2x-\cos^2 x$
	$\cos^2 x-\sin^2 x=\frac{1}{2}(2\sin x\cos x)-\cos^2 x$	✓ $\sin 2x=2\sin x\cos x$
	$\cos^2 x-\sin^2 x=\sin x\cos x-\cos^2 x$	
	$\cos^2 x-\sin^2 x-\sin x\cos x+\cos^2 x=0$	
	$(\cos x-\sin x)(\cos x+\sin x)+\cos x(\cos x-\sin x)=0$	✓ factors
	$(\cos x-\sin x)(\cos x+\sin x+\cos x)=0$	✓ factors
	$\cos x=\sin x$ or $2\cos x=-\sin x$ $\tan x=1$ $\tan x=-2$ ref $\angle = 45°$ ref $\angle = 63,43°$ $x\neq 45°+k.180°;k\in Z$ $x=116,57°+k.180°$	✓ both equations ✓ $x=116,57°$ ✓ $116,57°+k.180°;k\in Z$
		(7)
		[13]

QUESTION/*VRAAG* 7

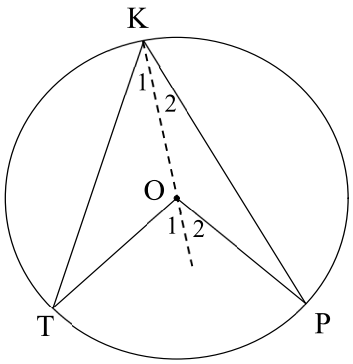
7.1	$180°$	✓ answer (1)
7.2		✓ asymptotes ✓ shape ✓ intercepts with axes (3)
7.3	$f(x)=\cos 2x$ $h(x)=\cos 2(x+45°)$ $=\cos(2x+90°)$ $=-\sin 2x$	✓ answer (1)
7.4	$y\in[-1;1]$ **OR** $-1\le y\le 1$	✓ $y\in[-1;1]$ (1) ✓ $-1\le y\le 1$ (1)
7.5	$\tan 2x-1=0$ $\tan 2x=1$ $2x=45°$ $x=22,5°$ $(1-\tan 2x)(\cos 2x)\ge 0$ $-(\tan 2x-1)(\cos 2x)\ge 0$ $(\tan 2x-1)(\cos 2x)\le 0$ $x\in[0°;22,5°]\cup[112,5°;135°)$ **OR** $0°\le x\le 22,5°$ or $112,5°\le x<135°$	✓ $x=22,5°$ ✓ $(\tan 2x-1)(\cos 2x)\le 0$ ✓ first interval ✓ second interval (4)
		[10]

QUESTION/*VRAAG* 8



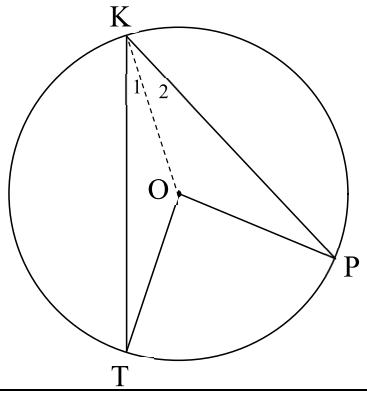
8.1	Area of ABCD = $BC \times AB$ $648 = 2AB \times AB$ $AB^2 = 324$ $AB = 18$ cm	✓ $BC = 2AB$ ✓ substitution into area of rectangle (2)
8.2	$AC^2 = AB^2 + BC^2$ [Pythagoras] $= 18^2 + 36^2$ $AC = \sqrt{1620} = 18\sqrt{5} = 40,25$ cm	✓ $AC^2 = 18^2 + 36^2$ ✓ answer (2)
8.3	$\frac{KC}{\sin K\hat{B}C} = \frac{BC}{\sin B\hat{K}C}$ $\frac{KC}{\sin 104°} = \frac{36}{\sin 52,6°}$ $KC = \frac{36 \sin 104°}{\sin 52,6°}$ $KC = 43,97$ cm	✓ substitution into sine rule ✓ answer (2)
8.4	$AK^2 = AB^2 + BK^2$ [Pythagoras] $= 18^2 + 18^2$ $AK = \sqrt{648} = 18\sqrt{2}$ cm $= 25,46$ cm $KC^2 = AK^2 + AC^2 - 2AK.AC \cos K\hat{A}C$ $(43,97)^2 = (18\sqrt{2})^2 + (18\sqrt{5})^2 - 2(18\sqrt{2})(18\sqrt{5})(\cos K\hat{A}C)$ $\cos K\hat{A}C = 0,16...$ $K\hat{A}C = 80,60°$	✓ length of AK ✓ substitution into cosine rule ✓ simplification ✓ answer (4)
		[10]

QUESTION/VRAAG 9



9.1	Construction: Draw KO produced	✓ construction
	$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ/*buite ∠ van* Δ]	
	But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ S / R
	$\therefore \hat{O}_1 = 2\hat{K}_1$	✓ S
	$\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ/*buite ∠ van* Δ]	
	But $\hat{K}_2 = P$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	
	$\therefore \hat{O}_2 = 2\hat{K}_2$	✓ S
	$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$	✓ S
	$= 2\left(\hat{K}_1 + \hat{K}_2\right)$	
	$\therefore T\hat{O}P = 2\ T\hat{K}P$	(5)

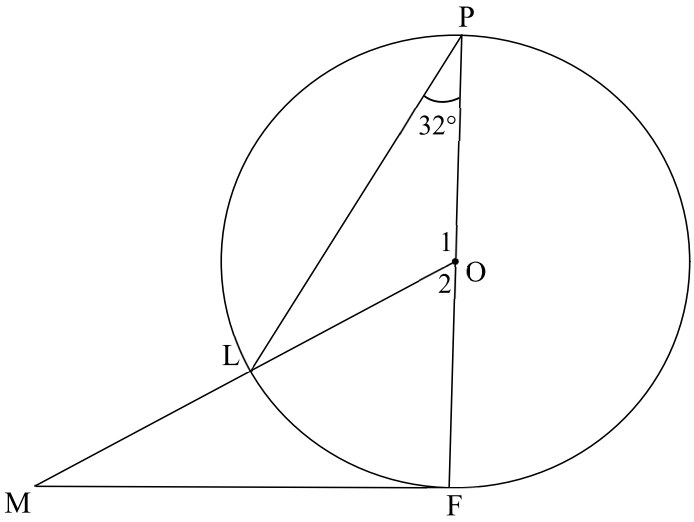
OR



9.1	Construction: Draw KO	✓ construction
	$\hat{T} = \hat{K}_1$ [∠ s opp. equal sides/*∠e teenoor gelyke sye*]	✓ S / R
	$\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of ∠ s of Δ /*binne ∠e van* Δ]	✓ S
	$\hat{P} = \hat{K}_2$ [∠ s opp. equal sides/*∠e teenoor gelyke sye*]	✓ S
	$\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of ∠ s of Δ / *binne ∠e van* Δ]	✓ S
	$T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [∠ s around a point/*∠e om 'n punt*]	
	$= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$	
	$= 2\hat{K}_1 + 2\hat{K}_2$	
	$= 2\left(\hat{K}_1 + \hat{K}_2\right)$	(5)

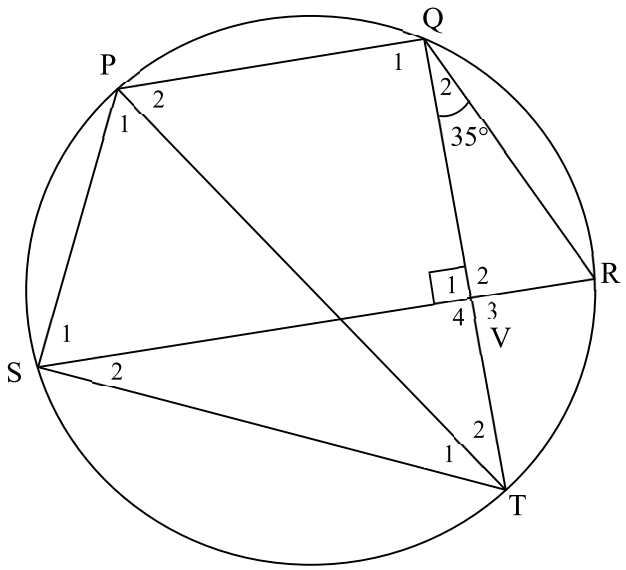
	$\therefore T\hat{O}P = 2\,T\hat{K}P$	

9.2



9.2.1	$\hat{O}_2 = 64°$	[∠ at centre = 2 ×∠ at circumference/ *Midpts∠ = 2 × Omtreks∠*]	✓ S ✓ R (2)
	OR		
	PLO = $\hat{P}$ = 32°	[∠s opp equal radii/ *∠e teenoor gelyke radiusse*]	✓ S / R
	$\hat{O}_2 = 64°$	[ext ∠ of Δ / *buite ∠ van Δ*]	✓ S (2)
9.2.2	$P\hat{F}M = 90°$	[tan ⊥ diameter/*raaklyn ⊥ middellyn*]	✓ S ✓ R
	$\hat{M} = 26°$	[sum of ∠ s of Δ/*binne ∠e van Δ*]	✓ S (3)
			[10]

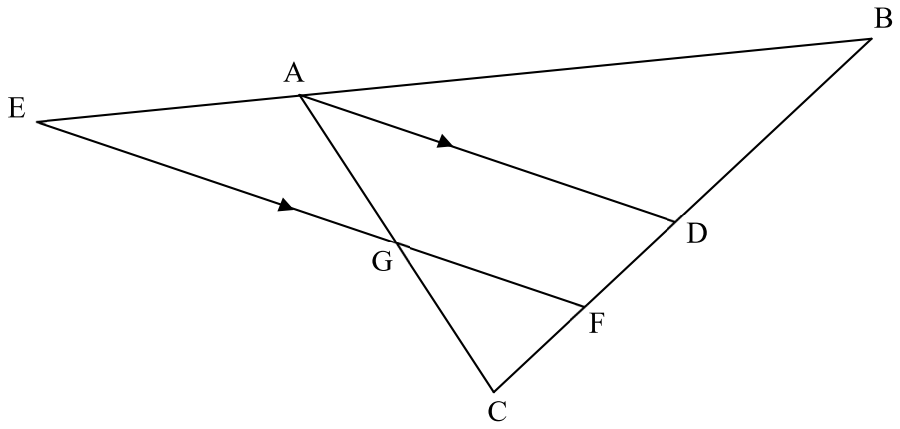
QUESTION/VRAAG 10



10.1	$\hat{R}=55°$ [sum of ∠s in Δ/*binne ∠e van Δ*] $\therefore Q\hat{T}S=55°$ [∠s in the same seg/*∠e in dieselfde segment*] **OR** $\hat{S}_2=35°$ [∠s in the same seg/*∠e in dieselfde segment*] $\therefore Q\hat{T}S=55°$ [sum of ∠s in Δ/*binne ∠e van Δ*]	✓ S ✓ S ✓ R (3) ✓ S ✓ R ✓ S (3)
10.2	$S\hat{P}Q=125°$ [opp ∠s of cyclic quad/teenoorst. *∠e van kvh*] $\hat{S}_1=\hat{R}=55°$ [given/*gegee*] $S\hat{P}Q+\hat{S}_1=180°$ $\therefore PQ \parallel SR$ [co-int ∠s supp/*ko-binne ∠e suppl*] **OR** $\hat{S}_1=\hat{R}=55°$ [given/*gegee*] $P\hat{Q}R=125°$ [opp ∠s of cyclic quad/teenoorst. *∠e van kvh*] $\therefore \hat{Q}_1=125°-35°=90°$ $\therefore \hat{Q}_1+\hat{V}_1=180°$ $\therefore PQ \parallel SR$ [co-int ∠s supp/*ko-binne ∠e suppl*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)

10.3	$\hat{Q}_1 = 90°$	[co-int ∠s; PQ|| SR/ *ko-binne ∠e; PQ ||SR*]	✓ S
	∴ PT is a diameter	[converse ∠ in semi-circle/ chord subtends 90°∠ *omgekeerde ∠ in halwe sirkel / koord onderspan* 90° ∠]	✓ R (2)
	OR		
	$\hat{S}_2 = 35°$	[ext ∠ of ΔSVT or sum of ∠s in Δ *buite ∠ v* Δ of *binne ∠e van* Δ]	✓ S
	$P\hat{S}T = 90°$		
	∴ PT is a diameter	[converse ∠ in semi-circle/ chord subtends 90°∠ *omgekeerde ∠ in halwe sirkel / koord onderspan* 90° ∠]	✓ R (2)
			[8]

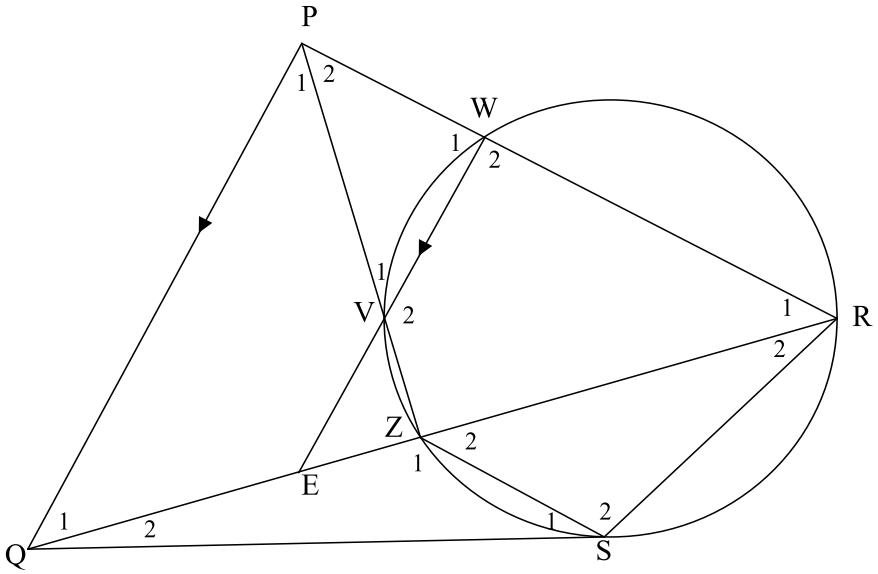
QUESTION/*VRAAG* 11



11.1.1	$\frac{FD}{CF}=\frac{GA}{CG}$ [prop theorem; AD || EF/line || one side of Δ/ *eweredigheidst.; AD || EF / lyn || een sy v Δ*] $\frac{FD}{CF}=\frac{2}{3}$	✓ S ✓ answer (2)
11.1.2	$FD=\frac{2}{3}CF$ $FD=\frac{2}{3}(2x)=\frac{4}{3}x$ $\frac{BA}{EA}=\frac{BD}{FD}$ [prop theorem; AD || EF/line || one side of Δ/ *eweredigheidst.; AD || EF / lyn || een sy v Δ*] $\frac{BA}{EA}=\frac{5x-\frac{4}{3}x}{\frac{4}{3}x}$ $=\frac{11}{3}\times\frac{3}{4}$ $=\frac{11}{4}$	✓ $\frac{4}{3}x$ ✓ S ✓ substitution ✓ answer (4)

11.1.3	$\frac{\text{Area of } \Delta GCF}{\text{Area of GFDA}} = \frac{\text{Area } \Delta GCF}{\text{Area } \Delta CDA - \text{Area } \Delta GCF}$ $= \frac{\frac{1}{2}GC.CF\sin\hat{C}}{\frac{1}{2}AC.CD\sin\hat{C} - \frac{1}{2}GC.CF\sin\hat{C}}$ $= \frac{\frac{1}{2}(3k)(3p)(\sin\hat{C})}{\frac{1}{2}(5k)(5p)(\sin\hat{C}) - \frac{1}{2}(3k)(3p)(\sin\hat{C})}$ $= \frac{\frac{1}{2}(9kp)(\sin\hat{C})}{\frac{1}{2}\sin\hat{C}(25kp - 9kp)}$ $= \frac{9}{16}$	✓ GFDA = ΔCDA – ΔCGF ✓ $\frac{1}{2}(GC)(FC)\sin\hat{C}$ ✓ $\frac{1}{2}AC.CD\sin\hat{C} - \frac{1}{2}GC.CF\sin\hat{C}$ ✓ answer (4)

11.2



11.2.1	$\frac{QE}{QR}=\frac{PW}{PR}$ [prop theorem; PQ || WE/line || one side of Δ / *eweredigheidst.; PQ || WE / lyn || een sy v Δ*] $PR=\frac{PW.QR}{QE}$	✓ S ✓ R (2)
11.2.2	$\frac{PQ}{RQ}=\frac{QZ}{QP}$ $[\Delta PQZ \text{ ||| } \Delta RQP]$ $\therefore PQ^2 = RQ.QZ$	✓ $\frac{PQ}{RQ}=\frac{QZ}{QP}$ (1)
11.2.3	In ΔQSZ and ΔQRS $\hat{Q}_2=\hat{Q}_2$ [common ∠ / *gemeenskaplike∠*] $\hat{S}_1=\hat{R}_2$ [tan chord theorem/*raaklyn koord stelling*] $\hat{Z}_1=Q\hat{S}R$ [3rd ∠ of Δ] $\therefore \Delta QSZ \text{ ||| } \Delta QRS$ [∠∠∠]	✓ S ✓ S/R ✓ S **OR** R (3)
11.2.4	$\frac{QS}{QR}=\frac{QZ}{QS}$ [ΔQSZ|||ΔQRS] $\therefore QS^2 = QZ.QR$ But $PQ^2 = RQ.QZ$ [proved in 11.2.2] $\therefore PQ = QS$	✓ S / R ✓ S ✓ S (3)

11.2.5	$\frac{PQ}{RQ} = \frac{PZ}{PR}$ $[\Delta PQZ \| \| \| \Delta RQP]$ $PR = \frac{QR.PZ}{PQ}$ $PR = \frac{PW.QR}{QE}$ [proved in 11.2.1] $\therefore \frac{PW.QR}{QE} = \frac{QR.PZ}{PQ}$ $PW = \frac{QE.PZ}{PQ}$ But $PQ^2 = RQ.QZ$ [proved in 11.2.2] $\therefore PQ = \sqrt{RQ.QZ}$ $\therefore PW = \frac{QE.PZ}{\sqrt{QR.QZ}}$	✓ $PR = \frac{QR.PZ}{PQ}$ ✓ S ✓ $PW = \frac{QE.PZ}{PQ}$ ✓ $PQ = \sqrt{RQ.QZ}$ (4)
		[23]

TOTAL/*TOTAAL*: 150